THE INVERSE AC JOSEPHSON EFFECT IN JOSEPHSON TUNNEL JUNCTIONS AT ENERGY GAP FREQUENCIES

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On the basis of the Werthamer equation it is shown that the inverse ac Josephson effect is possible in tunnel junctions at much higher frequencies than predicted by the RSJ model.

Since Levinsen (1) has proposed a voltage standard realization on the basis of the inverse ac Josephson effect, there are extensive investigations dealing with this subject (2-4). Within the frame of the RSJ model (5), Kautz (2) showed that the inverse ac effect is possible only for hysteretic junctions (McCumber parameter $\beta \gg 1$) and at low external frequencies ($h \cdot f \lesssim 1.16eR_N I$). The second limitation is mainly caused by the simple linear Ohmic current component in the resistively shunted junction. Unfortunately, the low-frequency region in hysteretic junctions shows significant indications of chaotic behaviour. Consequently, there is a great sensitivity of the effect, e.g. to small variations of the intensity of the external radiation.

High-capacitance tunnel junctions, however, reveal a highly nonlinear quasiparticle current component and low excess current for voltages within the gap region. Hence, the inverse ac Josephson effect should be observed at higher frequencies, too. For quantitative examination of this assumption we considered the Werthamer equation (6,7)

$$\begin{split} i(s) &= \frac{g^2}{4} \beta_c \ddot{x}(s) -g \int_0^\infty ds'(p(s')sin\frac{1}{2}(x(s-s')+x(s)) \\ &+ q(s')sin \frac{1}{2}(x(s-s')-x(s))) \\ &= i_o + i_1 sin(ws) . \end{split}$$

The real retardation functions p(s'), q(s') give the solid-state properties of the tunnel junction, x(s) is the phase difference depending on the normalized time $s = t(\Delta_1 + \Delta_2)/\hbar$, and $w = hf/(\Delta_1 + \Delta_2)$ is the dimensionless frequency of the external ac bias current. Furthermore, we have introduced a constant $g = (\Delta_1 + \Delta_2)/eR_NI_c$ depending on the energy gaps at zero temperature of both superconducting electrodes.

We anticipate a periodic solution which obeys the relation

$$x(s + \frac{1}{m}2\pi m) = x(s) + 2\pi l$$
, (2)

i.e. we make the ansatz

$$x(s) = x_{o} + \frac{1}{m}ws + \sum_{k=1}^{\infty} a_{k}sin(\frac{k}{m}ws+b_{k})$$
(3)

with the unknown constants x_0 , a_k , b_k . An approximate solution of the corresponding nonlinear algebraic equation system in the Fourier space for small values of the parameter

$$p_{0} = 1/g \beta_{c} w^{2} \ll 1$$
 (4)

can be found (8). In this case, the dc current through the junction at the Josephson step with the mean voltage (I/m)(w/2) consists of three terms:

$$o_{\text{out opa}}^{(1,m)} = i_{\text{out opa}}^{+i} (i_1) + i_{\text{ostep}}^{-(i_1,x_0)}.$$
(5)

Here we only represent the results up to the first order in p_{o} and for I = m = 1:

$$= g(\operatorname{Im} i_q(\frac{w}{2}) + 2p_o \operatorname{Re} i_p(\frac{w}{2}) \operatorname{Im} i_p(\frac{w}{2}))$$

$$i_{ostep} = 2p_{o}Re i_{p}(\frac{w}{2}) i_{1} sin(x_{o}).$$
(6)

i (w) and i (w) are the Fourier amplitudes of p(s) and q(s), respectively, using the sign convention of Poulson (9). The first term of eq. (5) in general contains the autonomous excess current contributions (i₁=0) and is identical with Schlup's result (10). The second term, arising from the non-linearity of the current amplitude i (w), describes a photon-assisted tunneling process and remains zero in the lowest order of p . The third term depends on the absolute phase \times and produces the current step, which reaches the axis i_=0 if

$$\frac{2}{g\beta_c} \frac{i}{w^2} \operatorname{Re} i_p(\frac{w}{2}) \ge i_{oaut} + i_{opa}(i_1) \quad . \tag{7}$$

Contrary to the RSJ model this condition can be satisfied at high frequencies, too. In fig. 1 we compare the numerical solution of the eqs. (1) and (3) with the corresponding RSJ result.



FIGURE 1

Numerically calculated height of the first Josephson step (I=m=1) as a function of i1. The thick line represents the solution of the Werthamer equation with identical BCS electrodes at zero temperature, the thin line gives the corresponding RSJ result. The broken lines show the i₁-dependence of the step centre (enlarged by factor 2) in both models. At the value i ≈ 16 the periodic solution does not exist for all x (8). Junction parameters: $\beta_c = 10$, w = 0.9 and $g = 4/\pi$.

In the case of BCS electrodes at zero temperature the inverse ac Josephson effect is possible for parameters

$$0.2 \neq i_1 \neq 9.7$$
, (8)

whereas in the RSJ model the effect only can occur for i ≈ 6.5. An additional advantage is that the

effect remains stable within a great range of the interval (8). The optimum value for the normalized frequency w is determined by eq. (7). The right hand side should not be too large and the height of the first Josephson step must be as large as possible. Thus we get the conditions w < 2and w ≈ 1. Recent experimental results of Danchi, Habbal and Tinkham (11) seem to confirm our conclusions, though the authors did not mention the inverse ac Josephson effect in their paper.

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