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On the Sensitivity of Parameterized Convection to the Rate of Decay of Internal Heat Sources

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Thermal histories for the earth based on parameterized mantle convection models have previously indicated that the earth departs significantly from a steady thermal state. The non-steady state is manifest by a present-day excess of heat loss over heat production. We investigate the dependence of the heat production/heat loss ratio and mantle viscosity on heat source decay rate by varying the relative and absolute concentrations of the heat-producing isotopes of K, U, and Th in the mantle. We examine three models with progressively increasing effective half-lives, respectively characterized by K/U ratios of 6×10^4 ("chondritic"), 1×10^4 ("terrestrial"), and 0.25×10^4 ("low K/U"). Each of these models can be constrained to yield a common present-day heat flow, mantle temperature, and mantle viscosity, thus demonstrating the inability of those present-day constraints to differentiate between widely varying blends of the earth's nuclear fuel. The present-day heat production/heat loss ratio ranges from about 73% for the chondritic model to about 86% for the low K/U model. Other possible criteria diagnostic of the earth's radioisotope mix are the present-day isotopic abundances. The low K/U model requires U and Th greater by a factor of 2.5 over the chondritic model and K less by a factor of 10. However, measurements of concentrations in candidate mantle rocks reveal a range that far exceeds the relatively small variations that distinguish the different compositional models. Paleothermal conditions also may be indicative of the earth's radioisotope blend. The thermal state in the Archean calculated for the low K/U earth model is characterized by a mean mantle temperature about 100°C hotter and heat flow about 2 times greater than the present-day, whereas a chondritic earth model yields a mantle temperature 300°C hotter and heat flow 4 times greater. However, current estimates of paleo-heat flow and mantle melting temperatures are burdened with uncertainties sufficiently large to preclude their use as criteria diagnostic of the earth's radioisotope endowment.

INTRODUCTION

The thermal evolution of the earth has been controlled by the balance between heat production within the interior and heat loss through the surface. In a nonsteady thermal state, conservation of energy requires that the inequality between heat generation and heat loss be accounted for by a change in heat stored within the earth. Decay of the earth's internal heat sources, coupled with the finite transit time of heat escaping from the interior to the surface, necessarily results in a surface heat loss in excess of instantaneous heat production and therefore in a cooling earth [*Daly,* 1980].

The magnitude of this departure from a steady state has been a topic of interest for many years. *Urey* [1956] and *Birch* [1958] pointed out that the rate of heat loss from the earth, as it was known at that time, was similar to the rate of heat production in chondritic meteorites. This "chondritic coincidence" led to the widely accepted view of a chondritic earth in a very nearly steady thermal state. A dynamic basis for a nearly steady state was provided by *Tozer* [1965, 1972], who argued that the exponential dependence of mantle viscosity on temperature would have a powerful thermostatic effect, maintaining the mantle temperature at a value just sufficient to drive moderate convection.

More recent estimates of the heat loss from the earth incorporate revised calculations of the heat loss from oceanic regions and are about 30% higher than would be expected for steady state heat flow from a chondritic earth. The heat loss is now estimated to be about 40×10^{12} W [*Davies,* 1980*a; Sclater et al.*, 1980; *Pollack,* 1980] equivalent to an average heat production of 6.7×10^{-12} W kg^{-1} for the entire earth. This requires a significant departure from either a chondritic bulk composition or a steady thermal state, or both. *Davies* [1980*b*], *Schubert et al.* [1980], and *Stacey* [1980] have used convective thermal history models to investigate the present-day cooling rate. Each concluded that the present rate of internal heat production is substantially less, perhaps only one half to three fourths, the rate of heat loss and that the balance of the heat flux is due to cooling of the earth. Our purpose in this paper is to examine the dependence of the heat production/heat loss ratio, mantle viscosity and mantle cooling rate on the rate of decay of the radiogenic heat sources in the earth's interior.

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PARAMETERIZED CONVECTION MODELING

Parameterized modeling of mantle convection has served as the basis for a number of theoretical studies of the thermal evolution of the earth and terrestrial planets [*Sharpe and Peltier,* 1978, 1979; *Turcotte et al.*, 1979; *Schubert et al.*, 1979, 1980; *Davies,* 1980*b*; *Cook and Turcotte,* 1981]. A detailed development of the method can be found in the review paper by *Schubert* [1979]; we will only summarize it briefly here. The method makes use of an experimentally and theoretically based relationship between the heat flux out of a convecting layer and the temperature difference driving the convection. The relation is expressed in terms of two dimensionless parameters that describe convecting systems, the Rayleigh number (*Ra*) and the Nusselt number (*Nu*). For a horizontal layer heated from below the Rayleigh number is defined as

$$Ra = \alpha g(T-T_S)D^3/(\kappa\nu) \tag{1}$$

where T is the temperature at the base of the layer, T_S is the temperature at the upper surface, and the rest of the symbols are defined in Table 1. The Rayleigh number has a slightly different form for a layer heated from within, but *Schubert et al.* [1980] have justified the use of (1) for this case, with the provision that T represents the characteristic temperature of the convecting region. The viscosity of the layer appears in the denominator of (1) and is an exponential function of inverse temperature:

$$\nu = \bar{\nu} \exp (A/T) \tag{2}$$

McKenzie and Richter [1981] have criticized parameterizations which include temperature-dependent viscosity, but we have retained this effect in our calculations in order to be able to compare our results directly with those of *Schubert et al.* [1980]. The Nusselt number is the ratio

TABLE 1. Parameter Nominal Values

Parameter	Definition	Value
g	gravitational acceleration	9.8 m s^{-2}
α	thermal expansivity	$3 \times 10^{-5} \text{ K}^{-1}$
κ	thermal diffusivity	$10^{-6} \text{ m}^2 \text{ s}^{-1}$
D	depth of convecting layer	2.8×10^6 m
$\bar{\nu}$	minimum viscosity	$2.21 \times 10^7 \text{ m}^2 \text{ s}^{-1}$
A	activation temperature	5.6×10^4 °K
k	thermal conductivity	$4.2 \text{ W m}^{-1} \text{ K}^{-1}$
β	exponent in Nusselt-Rayleigh relation	0.3
ρc	volumetric specific heat	$4.2 \times 10^6 \text{ J m}^{-3} \text{ K}^{-1}$
R_c	core radius	3471 km
R_m	mantle outer radius	6271 km
Ra_c	critical Rayleigh number	1100
T_s	surface temperature	273°K

of the total heat flux to the conductive heat flux out of a layer:

$$Nu = (q_{cond} + q_{conv})/q_{cond} = qD/k\Delta T \tag{3}$$

The relation between these parameters which is exploited in parameterized convection models is of the form

$$Nu = (Ra/Ra_c)^\beta \tag{4}$$

where β is a constant. The theoretical and experimental basis for this relation has been reviewed by *Schubert* [1979]. For a wide range of values of Nu, Ra, and Ra_c, β has been found to maintain a value of about 0.3. Substitution of (4) into (3) gives an expression for the heat flux out of a convecting layer:

$$q = (Ra/Ra_c)^\beta k\Delta T/D \tag{5}$$

A heat balance for the mantle can be written

$$\rho c(R_m^3 - R_c^3)\dot{T} = -3qR_m^2 + Q(R_m^3 - R_c^3) \tag{6}$$

This expression neglects the heat flux out of the core and into the base of the mantle. *Stacey* [1980] has argued that core heat is transported by narrow plumes and that thermal convection in the mantle is largely unaffected by heating from below. The rate of radiogenic heat production Q is the sum of contributions from each of the four major heat-producing isotopes, ^{40}K, ^{238}U, ^{235}U, and ^{232}Th. It decays with time according to

$$Q = Q_0 \exp(-\bar{\lambda}t) \tag{7}$$

where $\bar{\lambda}$ is an effective decay constant determined by the relative proportions of the isotopes. Substitution of (5), (1), (2), and (7) into (6) provides a first-order nonlinear differential equation for the variation of mantle temperature with time [*Schubert et al.*, 1980].

We wish to investigate the effects of a variable effective decay constant on the inequality between present-day heat production and heat loss and on the thermal history of the earth. We consider three compositional models, each with different effective rates of decay, as summarized in Table 2. For purposes of comparison we will also consider the case of a non-decaying internal heat source.

SECULAR DECREASE IN RADIOGENIC HEAT PRODUCTION

The effective decay constant for a mixture of radiogenic heat sources with different individual decay constants is given by

$$\bar{\lambda} = \frac{\lambda_1 Q_{10}\exp(-\lambda_1 t) + \lambda_2 Q_{20}\exp(-\lambda_2 t) + \ldots}{Q_{10}\exp(-\lambda_1 t) + Q_{20}\exp(-\lambda_2 t) + \ldots} \tag{8}$$

where λ_1, λ_2, ... are the individual decay constants and Q_{10}, Q_{20}, ... are the respective heat production rates at time $t = 0$ [*Stacey*, 1980]. The parameter $\bar{\lambda}$ is a function of the relative abundances of the heat-producing isotopes and therefore variable in time.

We assume respective values of 0.997 and 0.003 for the present-day mass ratios of ^{238}U/U and ^{235}U/U. For the ratio ^{232}Th/U we use the value 3.5, and for ^{40}K/K we assume a value of 1.2×10^{-4} [*Stacey*, 1977*a*]. With these proportions established, specification of the ratio K/U is all that is necessary to characterize the relative abundances of the heat-producing radioisotopes and hence the effective decay constant for the mantle.

The mass ratio K/U in terrestrial crustal rocks has a nearly uniform value of about 1×10^4 [*Taylor and McLennan*, 1981]. In chondritic meteorites the ratio is somewhat higher, with a range of values from 2×10^4 to about 7×10^4 [*Eldridge et al.*, 1974]. For the mantle the K/U ratio is poorly known; estimates range from about 0.15×10^4 to 8×10^4. The low estimate is based on high ^{4}He/^{40}Ar ratios in glassy submarine basalts [*Fisher*, 1975]. A similar estimate was obtained by *Stacey* [1980] on the basis of atmospheric ^{40}Ar content. The higher estimates simply represent a terrestrial or chondritic value of the K/U mass ratio.

The significance of the K/U ratio lies in the large difference in their respective half-lives. ^{40}K has a half-life of 1.25 Ga, while ^{238}U and ^{232}Th have half-lives of 4.47 and 14.0 Ga, respectively. This means that with a high K/U fuel mix the rate of heat production would decrease by a factor of about 8 over the 4.5 Ga of earth history, while the rate of heat production for a low K/U mix would decrease only by about a factor

TABLE 2. Variation of Heat Production Through Time

	Chondritic	Terrestrial	Low K/U	Nondecaying
Present mass ratio K/U	6×10^4	1×10^4	0.25×10^4	—
Present mass ratio ^{40}K/^{238}U/^{235}U/^{232}Th	7.2/0.997/0.003/3.5	1.2/0.997/0.003/3.5	0.30/0.997/0.003/3.5	—
Present heat production ratio ^{40}K/^{238}U/^{235}U/^{232}Th	0.526/0.235/0.010/0.229	0.156/0.418/0.018/0.408	0.044/0.474/0.020/0.402	—
Present mean decay rate, yr^{-1}	3.36×10^{-10}	1.84×10^{-10}	1.39×10^{-10}	0
Present mean half-life, Ga	2.1	3.8	5.0	infinite
Decay rate averaged over 4.5 Ga, yr^{-1}	4.5×10^{-10}	3.4×10^{-10}	2.83×10^{-10}	0
Mean half-life over 4.5 Ga, Ga	1.5	2.1	2.6	infinite

Source *Stacey* [1977*a*,*b*]

of 3 over the same time interval. In the limiting case of nondecaying heat sources the earth would evolve from arbitrary initial conditions to a steady state determined by the uniform rate of heat production and the thermally activated rheology. It is reasonable, therefore, to suggest that a thermal history for the earth involving low K/U heat sources would lead to present-day conditions much closer to a steady state than would a history with high K/U heat sources.

CONVECTIVE THERMAL HISTORY MODELS

The characteristic temperature of the mantle through time can be obtained by numerical integration of the differential equation derived from (6), once an initial temperature has been specified. Heat of accretion and heat produced by segregation of the core were probably sufficient to bring the mantle to a high temperature very early in its history. In addition, substantial heat may have been liberated by decay of short-lived radioisotopes, particularly ^{26}Al [*O'Nions et al*, 1978]. However, a cold early history involving slow heterogeneous accretion has also been proposed [*Turekian and Clark*, 1969]. We have investigated a variety of initial temperatures to encompass these possibilities.

The two principal constraints that all numerical models must attempt to satisfy are the present-day heat flow from the mantle and the present-day viscosity of the mantle. The earth is presently discharging heat at a rate of about 40×10^{12}W [*Davies*, 1980*a*; *Sclater et al.* 1980; *Pollack*, 1980]. About 10% of this is attributable to radiogenic heat produced in the continental crust [*Pollack and Chapman*, 1977], leaving a heat flow from the mantle of 70-72 mW m^{-2}. Modeling of postglacial isostatic rebound for Fennoscandia and North America suggests a uniform Newtonian viscosity of 1 to 5×10^{21} Pa s for the entire mantle [*Cathles*, 1975; *Peltier*, 1976; *Wu and Peltier*, 1983]. For the mean density of the mantle, this corresponds to a kinematic viscosity of about 3×10^{17} m^2 s^{-1}. There are sufficient degrees of freedom in the choice of parameters to enable the present-day heat flow and viscosity constraints to be easily met.

Figure 1 illustrates thermal histories using two different initial temperatures, 2000° and 3200°K, for each of the compositional models. The value of the parameter Q_0 was iteratively adjusted in each case to yield a present-day mantle heat flow of 70 mW m^{-2}. All of the other parameters were held fixed at the values listed in Table 1. It is clear that

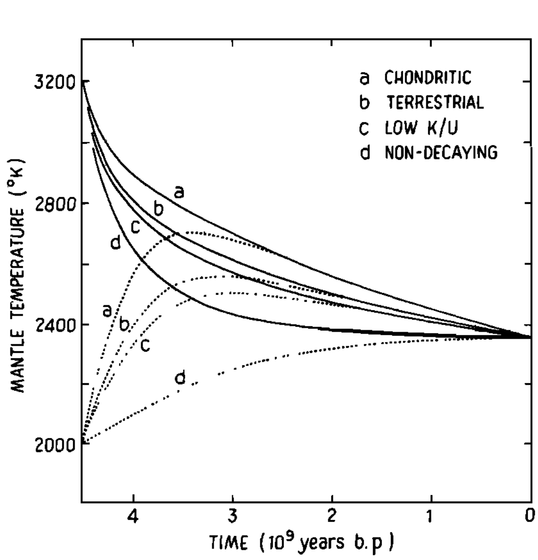

Fig. 1. Thermal histories for the earth using the nominal parameter values listed in Table 1 and the time-averaged decay constants listed in Table 2. Q_0 was adjusted in each case to yield a present-day heat flow from the mantle of 70 mW m^{-2}. Initial temperatures are 2000° and 3200°K, and the present-day temperature is 2445°K.

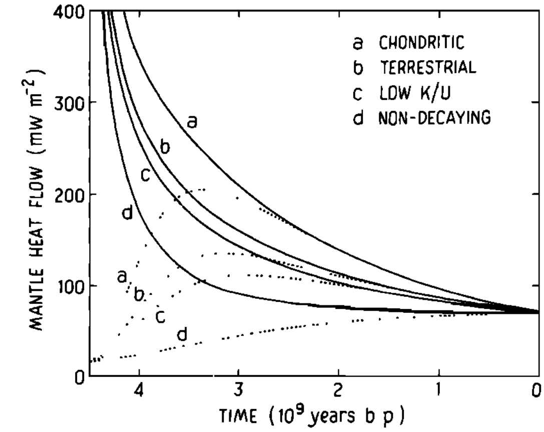

Fig. 2. Mantle heat flow corresponding to the thermal histories shown in Figure 1.

forcing each of the models to yield the same present-day heat flow also produces uniform present-day temperatures. This follows from (5) with fixed values of Ra_c and β. The value of the parameter A is the same as that used in the nominal model of *Schubert et al.* [1980], and has been chosen to yield a kinematic viscosity of 3×10^{17} m^2 s^{-1} at a temperature of 2500° [*Stacey*, 1977*b*]. Other combinations of $\bar{\nu}$ and A which meet this criterion produce thermal histories very similiar to those of Figure 1.

Each of the three compositional models produces a thermal history of the same form: an early stage in which the initial conditions play an important role, followed by an approximately linear decrease of temperature with time, with a characteristic cooling rate for each model. For a high initial temperature, vigorous convection cools the mantle rapidly and ensures that the memory of the initial condition is lost at an early stage for each compositional model. The fictitious model with nondecaying heat sources reaches a steady state about halfway through earth history. For a cold start, convection is sluggish, and the temperature initially rises according to the rate at which heat is produced. A chondritic mantle warms up relatively quickly and in less than 2 Ga establishes its characteristic cooling rate of 90°K Ga^{-1}. For the terrestrial model the initial conditions continue to be important for somewhat more than 2 Ga, after which the mantle cools at about 60°K Ga^{-1}. Early heat production for the low K/U model is considerably less than that for the other models, and the time required to establish the characteristic cooling rate of 30°K Ga^{-1} is nearly 3 Ga for an initial temperature of 2000°K. The fictitious model with nondecaying heat sources and a cold start warms slowly throughout earth history and approaches a steady state only at the present-day.

Figure 2 shows the mantle heat flow through time for each of the three compositional models and the fictitious model, corresponding to the high/low initial temperature thermal histories of Figure 1. The heat flow histories are similar in form to the temperature histories. The Archean heat flow values depend strongly on the mantle K/U ratio, ranging from about 2 to 4 times the present-day heat flux.

HEAT PRODUCTION/HEAT LOSS RATIO

Because of the enormous heat capacity of the mantle, even a very slow cooling rate provides a substantial heat flux through the outer surface of the earth. For the low K/U characteristic cooling rate of 30°K Ga^{-1}, heat withdrawn from the reservoir in excess of replacement heat production supplies about 14% of the present-day mantle heat flow. The higher terrestrial and chondritic cooling rates, 60° and 90°K Ga^{-1}, require withdrawals that supply 17 and 27%, respectively.

As noted above, cooling of the mantle occurs because the production

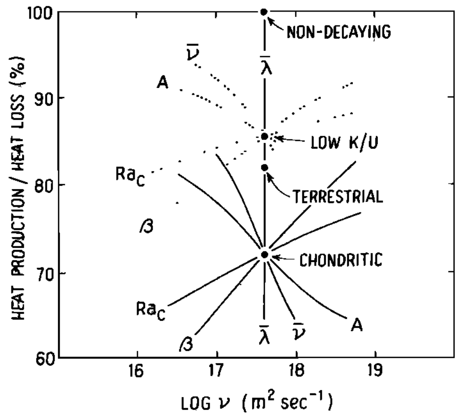

Fig. 3. Sensitivity diagrams [after *Schubert et al.*, 1980]. Solid and dotted curves are for chondritic and low K/U models, respectively. The central point common to the curves indicates the present-day mantle viscosity and heat production/heat loss ratio calculated using the nominal parameter values. The curves were generated by varying single parameters from the nominal case, and adjusting the value of Q_0 to yield the observed present-day heat flux. The relative sensitivities of present-day mantle viscosity and heat production/heat loss ratio to variations in single parameter values are shown by the slopes of the curves. Values of Ra_c, β, $\bar{\nu}$, and A at $\nu = 10^{17}$ m^2 s^{-1} are 6386, 0.265, 2.12 x 10^3 m^2 s^{-1} and 3.85 x 10^4 °K respectively; for $\nu = 10^{18}$ m^2 s^{-1} they are 392, 0.344, 3.10 x 10^9 m^2 s^{-1} and 6.85 x 10^4 °K.

of radiogenic heat within the volume of the mantle is less than the loss of heat at the surface. A convenient expression of this inequality is the heat production/heat loss ratio. Figure 3 illustrates the sensitivity of the heat production/heat loss ratio to variations in the values of single parameters, and the effect these variations have on calculated present-day viscosity. Following *Schubert et al.* [1980], we choose a nominal model, using parameter values listed in Table 1. (As noted earlier, the parameters of the nominal model yield a kinematic viscosity for the mantle of 3 x 10^{17} m^2 s^{-1}). We then generate suites of models in which a single parameter is allowed to vary from its nominal value, while continually adjusting Q_0 to maintain a present-day mantle heat flux of 70 mW m^{-2}. Two suites of models are shown by the curves in Figure 3, one with a chondritic nominal decay constant and the other with a low K/U decay constant. The curves of each suite intersect at a point corresponding to the arbitrarily chosen nominal model, and therefore the location of the intersection has no special significance. The significance of the curves lies in their slopes as they leave the common point. As observed by *Schubert et al.* [1980], varying the values of Ra_c, β, $\bar{\nu}$ and A can produce significant effects on the heat production/heat loss ratio but at the expense of departing from the present-day viscosity constraint.

However, for the thermal history models generated by varying $\bar{\lambda}$, the heat production/heat loss ratio is directly related to the effective half-life, and the present-day viscosity is completely insensitive to that parameter variation. This insensitivity is, in retrospect, an obvious and unavoidable outcome because holding the parameters Ra_c and β fixed insures that the uniform present-day heat flow corresponds to a uniform present-day temperature, as shown in Figure 1, and because holding the rheological parameters $\bar{\nu}$ and A fixed also, each model in this suite must yield exactly the same present-day viscosity, in good agreement with the value required by the isostatic uplift. Clearly then, all of the models in this group satisfy the present-day mantle heat flow and viscosity constraints, while yielding values from 73 to 86% for the ratio of heat generation to heat loss. With only modest relaxation of the present-day viscosity constraint, the ratio can exceed 90%. The fictitious model with nondecaying heat sources of course yields an equilibrium between heat production and heat loss.

The nominal model of *Schubert et al.* [1980] utilized a chondritic decay constant, and they did not include variations of the decay constant in their sensitivity investigations. They concluded that the heat production/heat loss ratio could not exceed 80%, but that conclusion can be seen as resulting in part from their choice of a chondritic decay constant; higher ratios result from the slower decay rates of the terrestrial and low K/U compositional models.

The heat production/heat loss ratio also appears to be somewhat model-dependent. *Stacey* [1980] used a different type of thermal history calculation based on the mechanical efficiency of convection. With a low K/U heat source model he found a maximum value for the present-day heat production/heat loss ratio of about 75%. *Davies* [1980*b*] used a different method of calculating thermal histories with parameterized convection and found values of 45% and 65%, using decay constants corresponding approximately to our chondritic and terrestrial models. We have found that substituting a low K/U decay constant into Davies' calculation gives a value of about 85%.

GEOCHEMICAL AND HISTORICAL DISCRIMINANTS

While the principal focus of this investigation has been on the effects of the heat source decay constant on the heat transfer process as expressed in the heat production/heat loss ratio, there are obvious geochemical implications which we will only briefly mention (see *Davies* [1980*b*], for a fuller discussion). The constraint of the present-day mantle heat flux, which all of our models meet, is computationally satisfied by adjusting the concentrations of the heat producing isotopes while maintaining the isotopic ratios appropriate to each compositional model. The resulting requisite concentrations of each isotope per unit mass of the mantle are shown in Table 3. If one assumes that the isotopes are distributed throughout the mantle and core, the concentrations per unit mass would be just two thirds of those shown in Table 3.

Unfortunately, there are few means by which independent estimates can be made of the concentrations of U and K in the mantle. A limited number of direct measurements of these concentrations in candidate mantle rocks have been made, and some of these are listed in Table 4. *Jagoutz et al.* [1979] analyzed the composition of spinel lherzolite xenoliths in alkali basalts from a large number of localities. Because these ultramafic nodules contained a full complement of basaltic elements, Jagoutz et al. argued that they should represent nearly undepleted mantle material. The uranium concentration in these rocks varied from 8 to 60 ppb, spanning the entire range of values in Table 3. The K/U ratios were all quite low, ranging from 365 to 2300. *Kramers* [1979] measured the K and U concentrations in clinopyroxene separates from various types of xenoliths in South African kimberlites. Again the variation in measured concentration covers the entire range of values in Table 3. The K/U ratios were variable, from a minimum of 2800 to a maximum of 1.3 x 10^5. Clearly, these data will not enable us to rule out any of the compositional heat source models.

Indirect estimates of the uranium concentration in the mantle are commonly based on the observation that the abundance ratios of refractory

TABLE 3. Mantle Concentrations of U, Th, and K and Present-Day Heat Production for Three Compositional Models

	U, ppb	Th, ppb	K, ppm	Present-Day Heat Production, 10^{12} W
Chondritic	16.5	57.8	990	26.0
Terrestrial	33.2	116.0	332	29.3
Low K/U	39.2	137.0	98	30.7

If isotopes were distributed throughout the earth, the concentrations would be two thirds of the mantle concentrations.

lithophile elements (Ca, Al, REE, Th, U, Sr) are nearly constant in a number of geologic environments as well as in chondrites [*Taylor*, 1979; *Sun*, 1982]. Mantle U is usually taken to be 2-3 times the chondritic composition, depending on the estimation of Ca and Al in the mantle. Mantle K is then computed from an assumed value of K/U [e.g., *Jochum et al.*, 1983]. Other methods of estimating the potassium content of the mantle are based on the isotopic composition of atmospheric argon [*Taylor*, 1979; *Sleep*, 1979; *Stacey*, 1980]. These calculations depend strongly on the degassing history assumed. For continuous degassing models, low potassium concentrations are required [*Sleep*, 1979; *Stacey*, 1980]. Table 4 also includes some of these inferred concentrations for comparison.

An alternative to present-day constraints can be sought in the thermal histories which are very different, particularly in the Archean and early Proterozoic, for each of the compositional models. The chondritic, terrestrial, and low K/U models predict temperatures at 1 Ga which are 300°, 200°, and 100° hotter than the present mantle temperature, respectively. Komatiites of about that age appear to have been extruded at temperatures as much as 300° higher than the hottest Phanerozoic extrusives [*Green*, 1981] and thus are consistent with the higher temperatures predicted by the chondritic heat source model. However, it is not clear whether the entire mantle was hotter by the same amount as the komatiite extrusives. *Smith* [1981] proposed a hotspot model for the origin of the komatiites, while *Green* [1981] emphasized that impact phenomena may have been important in the generation of these high-temperature magmas and that the characteristic upper mantle temperature may not have been as high as implied by the calculated komatiite extrusion temperatures. There seems to be enough uncertainty in the geochemical and tectonic setting of these ancient magma sources to admit also the possibility of the temperatures predicted by the heat source models with low K/U or terrestrial isotopic ratios.

TABLE 4. Estimated Mantle Concentrations of U and K

	U, ppb	K, ppm	Reference
Rock Type	Measured Values in Mantle-Derived Rocks		
Spinel-lherzolite nodules	8-60	10-138	*Jagoutz et al.* [1979]
Eclogite	8-30	151-1319	*Kramers* [1979]
Common peridotite	15-45	231-549	*Kramers* [1979]
Fertile peridotite	6-36	130-176	*Kramers* [1979]
Kimberlite cpx megacrysts	18-45	125-301	*Kramers* [1979]
Eclogite	43	330	*Tilton and Reed* [1963]
Peridotite	6	10	*Tilton and Reed* [1963]
	Geochemically Inferred Values		
Method			
Refractory element ratios	15-20	150-200	*Taylor* [1979]
Refractory element ratios	26	260	*Jagoutz et al.* [1979]
Refractory element ratios	21	267	*Jochum et al.* [1983]
Atmospheric argon		75-150	*Taylor* [1979]
Atmospheric argon	20	50	*Stacey* [1980]
Atmospheric argon		120 (mantle) -200 (core)	*Sleep* [1979]
Peridotitic komatiite partial melt model	27	230 (primitive mantle)	*Sun* [1982] and *Sun and Nesbitt* [1977]

CONCLUSIONS

The disparity between heat production and heat loss is the consequence of a competitive balance between convective efficiency and heat source decay rates. If heat transfer were instantaneous or if heat production were constant, a true equilibrium between heat production and heat loss would be maintained. Longer transit times for heat escaping from the earth's interior, or more rapid decrease in heat source strengths, result in greater differences between production and loss of heat. In parameterized thermal histories the convective efficiency is dependent on the parameters Ra_c and β as well as the rheological parameters A and $\bar{v}$. Consideration of this aspect of the problem has led *Schubert et al.* [1980] to the conclusion that a present-day heat production/heat loss ratio less than 75% cannot be avoided by any plausible model. We have found that decreasing the K/U ratio in the fuel can increase the heat production/heat loss ratio to about 85% while still meeting the present-day heat flow and mantle viscosity constraints. Reducing the K/U ratio below 0.25 x 10^4 has little additional effect on the heat production/heat loss ratio, however, because ^{235}U then becomes an important early heat source. Slight relaxation of the viscosity constraint with a low K/U fuel can yield a ratio exceeding 90%.

Schubert et al. [1980] also point out that several effects excluded from the calculations could lead to smaller heat production/heat loss ratios. The numerical models assume that heat loss from the core is insignificant, but as for the mantle, even a very slow cooling rate in the core would release substantial quantities of heat. Second, the models have assumed whole mantle convection. Conductive heat transfer through one or more internal thermal boundary layers in a stratified convective system would increase the transit time of heat to the surface [*McKenzie and Richter*, 1981]. Each of these effects could increase the difference between present-day heat production and loss and reduce the heat production/heat loss ratio.

Present-day constraints on heat flow, temperature, and viscosity for the mantle cannot discriminate between the compositional models we have examined. Moreover, while each compositional model implies a unique present-day concentration of U, Th, and K in the mantle, actual measurements of these isotopic abundances in candidate mantle rocks show a scatter that exceeds the variations implied by the models. Thus another possible present-day discriminant is in fact ineffective. The compositional models have marked historical differences in mean mantle temperature and cooling rate which in principle are also potential discriminants, if the geologic record were sufficiently unambiguous. Regrettably, the early and middle Precambrian record presently provides ample interpretive freedom to render it also an ineffective constraint on the earth's thermal history and radioisotope endowment.

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