Homework Assignment

1. Read and understand chapter 6-17 "Thermal Convection" and chapter 6-18 "Linear Stability Analysis" of TS†, pp. 272-279.

   (a) Derive the steady-state solution $T_c(y)$ to eq. (6-283), given by eq. (6-285). Why is this solution becoming unstable at large ΔT?

   (b) Explain the physical meaning of eqs. (6-289) to (6-292). How is the pressure term eliminated from these equations?

   (c) Explain the "mode selection" situation shown in Figure 6-38. What is the most rapidly growing wave disturbance and why?

   (d) Write a MATLAB code for this linear stability analysis; in particular, plot the dimensionless growth rate $\alpha' b^2/\kappa$ as a function of the disturbance wave number $2\pi b/\lambda$, similarly to (Fig. 6-38). How is the minimum value for the critical Rayleigh number determined?

   (e) Solve **Problem 6-27** at page 279 of TS†.

† Turcotte, D.L. and Schubert, G., 1982, Geodynamics, J. Wiley & Sons, New York

2. The thermal history of a terrestrial planet may be described in its simplest form by an ordinary differential equation for a supposed “average” mantle temperature T_M.

   McGovern & Schubert (1989) extended this simple concept by adding up the influence of a possible volatile exchange between atmosphere and planetary interior.

   (a) Read and understand the original paper (EPSL 96 (1989) 27-37).

   (b) In particular, write a MATLAB code to solve their eqs. (1) to (5), to derive their main results depicted in Figs. 4, 5, 6, 7 using a Runge-Kutta scheme (4th order). What is the meaning of the Urey ratio, and how is its time evolution ?

   (c) Explain the described “thermostat” regulation effect of viscosity on the secular cooling process.

3. Access to MATLAB is available either locally here at the CIP Pool of the chemistry building (Golm H25), or at the University network (campus license).

   This homework is due April 8, 2019 (deadline).