Convection modeling with citcom

1. Introduction

This tutorial is adapted from a CIDER tutorial given by Peter van Keken, Carolina Lithgow-Bertelloni, and Louise Kellogg. It will allow you to investigate some aspects of mantle convection modeling using the finite element code Citcom. We will first explore the installation and basic use of the code and associated applications before investigating the relationships between the Nusselt and Rayleigh numbers for isoviscous and (for the brave hearted) temperature-dependent viscosity.

2. Preliminaries

Citcom

Citcom is the name associated with a number of related finite element codes that found their origin at Caltech in the mid-90s. Louis Moresi is

the principle author of the version we will be using. You are welcome to keep a version of this code but – as Louis requests in the source code – please be respectful of the time that went into creating this code. The code solves the incompressible Boussinesq equations for mantle convection

$$\nabla \cdot \boldsymbol{v} = 0 \tag{1}$$

$$\nabla p = \nabla \cdot \boldsymbol{\sigma} - Ra\, T \cdot \vec{e_z} \tag{2}$$

$$\frac{\partial T}{\partial t} + \boldsymbol{v} \cdot \nabla T = \nabla^2 T + Q \tag{3}$$

where Ra is the Rayleigh number and Q is a non-dimensional number representing internal heating.

Download and installation

The code can be downloaded from
`http://www.dynamicearth.de/download/CIDER.tgz`

This file is a compressed tar file that on a Mac (or linux PC) should be automatically decompressed and the files should be installed in the Downloads section of your home directory.

To run Citcom and analyze the models you will need to use the Unix shell. Find the shell (pictured by a window with a cursor in it) on your toolbar or in Finder –> Applications. Start a shell by clicking on it. A window will appear with a prompt, probably of the form:

```
computer:name$
```

which is the command line interface.

We will shorten this full prompt to $ in the descriptions below. You can list files with "ls", change directory with "cd Desktop". Locate the CIDER directory and change into it. You should have a list of directories (such as Citcom.build.file, Perl_scripts, CIDER06). Change directories to CIDER06.

Here you should see a few example directories, including 00_test, a doc directory (which contains this document) and a number of executables (citcom, extract_Nu, extract_Vrms). The Macs in this computer pool should be able (under MacOSX) to run the citcom executable as it is. Check this by changing the directory to 00_test and running

```
$ ../citcom input-test
```

This program should terminate in less than a minute with a message about a ppm file being written and some statements about cpu timing. If there are error messages you may need to recompile citcom.
To recompile:

```
$ cd ~/Downloads/CIDER/Shell_scripts
$ build-citcom
$ cp cII* CIDER06/citcom
```

and rerun the code in the test directory.

3. Basic model setup

In all cases we will assume a 2D Cartesian geometry. The initial model has aspect ratio 1 and a fluid of uniform viscosity. The box is heated from below (fixed temperature 1) and cooled from above (fixed temperature 0). The side boundaries have reflecting (symmetry) boundary conditions. The boundaries of the bottom are stress-free (i.e., not rigid). The equations are solved by a multigrid method, which requires the discretization into a fairly regular grid. In these examples we will limit ourselves to a uniform discretization, with the initial model at 33×33 nodal points.

The initial condition for temperature is a conductive solution with a small (10%) harmonic perturbation that will cause to initiate convection when the Rayleigh number is large enough. You can also specify the name of a file that contains the initial condition (which we will use to restart convection runs). In the first few examples we will try to get a steady-state, which in this code is obtained by letting time integration

continue long enough. This works fine at moderate Ra, but becomes increasingly harder when the convection "wants" to be time-dependent.

4. First full convection run

Find the example directory 0_iso_1d4. Check out the input file "input-2d_Ra1e4". You can do this with any editor through MacOSX, or with vi or emacs from the shell. The top 10 lines set some variables which are defined as:

`rayleigh`	thermal Rayleigh number
`maxstep`	maximum number of time steps
`finetunedt`	the multiplication factor for the self-selecting timestep
`datafile`	common root of all output filenames
`storage_spacing`	store output files every ...
`write_spacing`	store output files every ...
`TDEPV`	indication whether temperature dependent viscosity is used

`rheology_T_type`   choice of viscosity function

`viscT1, viscN0`   parameters in viscosity function

`dimenx`   non-dimensional grid size of the mesh
                    in x-direction

`levels`   number of multigrid levels

`perturbk`   wave number of the initial perturbation

`previous_temperature_file`   name of file with initial condition

The last variable is commented out (with a hash mark at the start of the line). Remove this hash mark when you want to do a restart from an existing file. The rest of the file is better left unchanged.

Run the citcom code with this input file

```
$ ../citcom input-2d_Ra1e4
```

If you do not like the output rolling on the screen or would like to check things in this shell while the code runs you can redirect the output and run it in the background with

```
$ ../citcom input-2d_Ra1e4 &
```

Use "jobs" to check whether it is still running, "fg" to bring it to the foreground, "ctrl-C" to kill it.

The code will produce files ending in .horiz_ave, .node_data, and .ppm. The first contains, well, horizontal averages; the second contains the temperature in the nodes, and last one is a graphics file that can be converted to gif by a program called graphic_convector (which should be available on the Macs).

Some information about the run is printed in the logs ending in .log (which contains basic info about the convergence of the multi grid cycle, computational time used) and .timelogs (which prints model time, Nu, Vrms etc.).

We will extract time series Nu(t) and Vrms(t) from the latter one using the utilities ../extract_Nu and ../extract_Vrms; the time series can be plotted with xmgrace.

Graphics output: temperature

Citcom produces plots of the temperature field and surface velocity in PPM format. Open GraphicConverter to view these files (Finder –> Applications –> GraphicConverter). If all goes well you can navigate to the run directory and find images of the ppms. Click on the thumbnails for a larger view. The interface is not very "smart" (i.e., you will need to move out of and back into a directory to refresh the file list) but it works. In the images for the first run you should see a spin-up with diffusion slowly becoming more important in the middle of the box.

Graphics output: time series

If you have not done so already, run

```
$ ../extract_Nu
$ ../extract_Vrms
```

which generates the files Nu.dat and Vrms.dat.

You can inspect the final values by

```
$ tail Nu.dat Vrms.dat
```

the last values should be around 4.8 and 42. To plot these files you can import them in your favorite graph generator, or use Unix tools like xmgrace or gnuplot. You will need to run them (hopefully installed using the "fink" tool in /sw/bin/xmgrace resp. /sw/bin/gnuplot under MacOSX) in an X-windows terminal. Find the X-windows terminal in the toolbar or applications. Run from this window

```
$ xmgrace Nu.dat
```

or, resp., after starting gnuplot (the quotation marks are essential)

```
$ plot "Nu.dat"
```

You should see the spin-up and stabilization of the convection reflected in the time-series. Inspect the last 20 or so entries in Nu.dat; you may notice that the model has not gotten to steady-state yet. Fix this.

5. Exploring mantle convection

(1) Check out the input file in `1_iso_1d4_aspect2` to see how you set up the isoviscous problem in a box of aspect ratio 2. How do Nu and $Vrms$ change ? What is a good estimate for the effective aspect ratio of convection in the Earth ?

(2) Explore the variation of Nu and $Vrms$ with Ra for a reasonable range (around 10^4). Can you estimate the critical Rayleigh number ? Verify the accuracy of your results with the "best" answers in the thermal benchmark (Blankenbach et al., GJI, 1989):

```
Ra=10⁴:   Nu= 4.8844, Vrms= 42.865
Ra=10⁵:   Nu=10.5412, Vrms=193.215
Ra=10⁶:   Nu=21.9720, Vrms=833.990
```

What happens at the higher Rayleigh numbers in terms of accuracy of the solution ? Are there other diagnostics you could use to see there is numerical trouble ?

It may be quite hard to get steady state results. Refine the grids for those cases that you do not feel comfortable with by increasing the input parameter levels (note: increase by 1 causes doubling of number of grid points in each direction and an increase in computational time of at least 4 times).

(3) Explore the temperature dependence of viscosity as provided in 2_etaT_1d4. Note that there are two input files: one to set up an initial condition (input-iso_initial), and one to use the final result as input for the temperature-dependent case. To run this case:

```
$ ../citcom input-iso_initial
$ cp ra1e4-2d.00500.node_data init.node_data
$ ../citcom input-etaT_Ra1e4
```

The viscosity in this case is prescribed by

$$\eta(T) = \exp\left(-bT\right)$$

with the parameter b represented by `viscT1` in the input file.

The input file in this directory is set up for the Blankenbach et al. benchmark 2a (Ra=10^4, b = ln1000, which has "best" values of Nu=10.066 and Vrms=480.43).

Why are Nu and $Vrms$ in this case higher than that for the isoviscous case with the same Ra ?

Can you get accurate results at 33×33 ?

Reference

Blankenbach et al. (1989), *A benchmark comparison of mantle convection codes*, Geophys. J. Int., vol. 98, pp. $23 - 38$.

Appendix (I)

Rayleigh-Benard instability

We consider a fluid layer with depth b heated from below (see figure), its upper surface $z = -b/2$ is maintained at the relatively cold reference temperature T_0 and its lower boundary is kept at the relatively hot temperature T_1 ($T_1 > T_0$). We assume that there are no heat sources in the fluid.
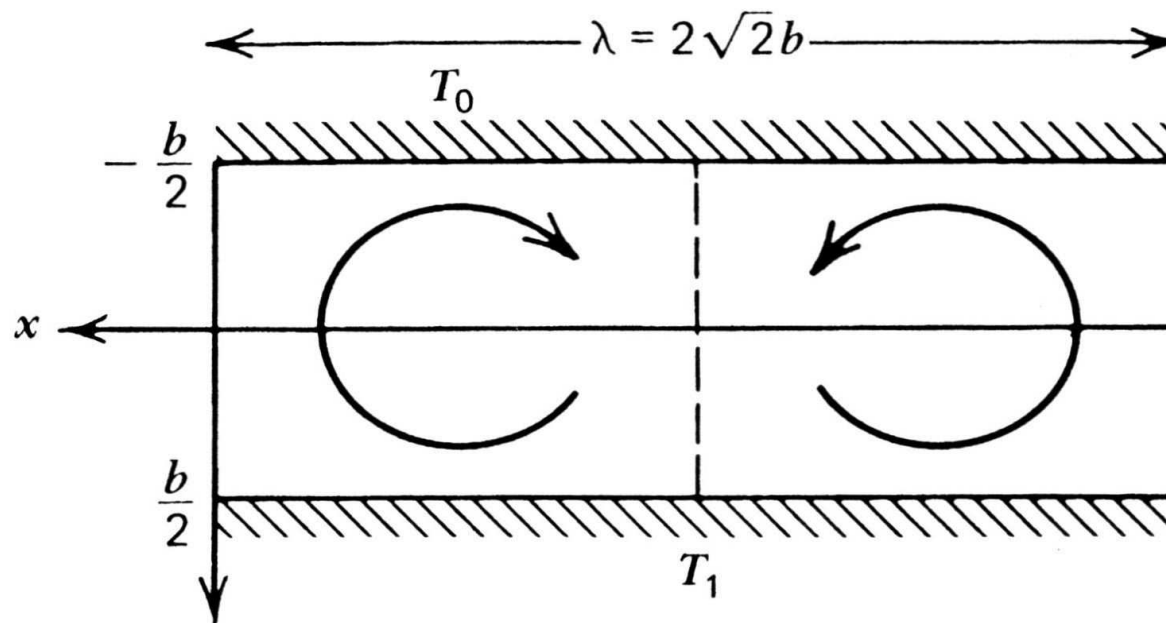
Buoyancy forces tend to drive convection in the fluid layer. Fluid near the heated lower boundary becomes hotter and lighter than the overlying fluid and tends to rise. Similarly, fluid near the colder, upper boundary is denser than the fluid below and tends to sink. However, the motion does not take place for small temperature differences across the layer because the viscous resistance of the medium to flow must be overcome.

In the absence of convection, that is, for $T_1 - T_0$ sufficiently small, the fluid is stationary ($v_x = v_z = 0$) and we can assume that a steady ($\partial T/\partial t = 0$) conductive reference state with ($\partial T/\partial x = 0$) exists. The temperature profile $T_{cond}(z)$ of this reference state follows therefore from

$$\frac{d^2 T_{cond}}{dz^2} = 0 \tag{4}$$

and is given with

$$T_{cond}(z) = \frac{T_1 + T_0}{2} + \frac{T_1 - T_0}{b} \cdot z \tag{5}$$

Conservation equations (2D)

Assuming a "Boussinesq approximation", the conservation equations for mass, energy and momentum (1) − (3) take the following form

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0 \tag{6}$$

$$\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_z \frac{\partial T}{\partial z} = \kappa \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right) \tag{7}$$

$$0 = \eta \left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial z^2} \right) - \frac{\partial p}{\partial x} \tag{8}$$

$$0 = \eta \left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial z^2} \right) - \frac{\partial p}{\partial z} + \rho_0 (1 - \alpha (T - T_0)) \, g \tag{9}$$

We now introduce the so-called *stream function* ψ according to

$$v_x = -\frac{\partial \psi}{\partial z}, \quad v_z = +\frac{\partial \psi}{\partial x} \tag{10}$$

The benefit of introducing the stream function is that we can eliminate the pressure term in the momentum equations (8) and (9) and automatically satisfy the mass balance, equation (6). Now we arrive at

$$\frac{\partial T}{\partial t} - \frac{\partial \psi}{\partial z}\frac{\partial T}{\partial x} + \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial z} = \kappa \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right) \tag{11}$$

$$0 = \eta \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right)^2 \psi - \rho_0 g \alpha \frac{\partial T}{\partial x} \tag{12}$$

Note that instead of solving 4 equations for 4 independent variables, we now have to solve 2 equations for 2 independent variables $T(x, z, t)$ and $\psi(x, z, t)$.

In addition, these solutions are subject to the following boundary conditions: We assume that the surfaces at $z = -b/2$ and $z = +b/2$ are isothermal and that there is nothing that exerts a shear stress on the fluid (this is called a "free surface" boundary condition).

Linear stability analysis

At the *onset of convection*, the fluid temperature is nearly the conduction temperature profile and deviations δT from T_{cond} are arbitrary small. Similarly, the convection velocities v_x and v_z are infinitesimal small when motion takes place first.

We can therefore write ($\psi_{cond} \equiv 0$)

$$T(x, z, t) = T_{cond}(z) + \delta T(x, z, t) \tag{13}$$

$$\psi(x, z, t) = \psi_{cond} \quad + \delta\psi(x, z, t) \tag{14}$$

and simplify the resulting equations to linear terms in δT and $\delta\psi$. It follows

$$\frac{\partial\,\delta T}{\partial t} + \frac{T_1 - T_0}{b}\frac{\partial\,\delta\psi}{\partial x} = \kappa\,\Delta\,\delta T \tag{15}$$

$$0 = \eta\,\Delta^2\,\delta\psi - \rho_0 g\alpha\frac{\partial\,\delta T}{\partial x} \tag{16}$$

Because these equations are linear equations with constant coefficients, we can solve them by the method of separation of variables. The boundary conditions are automatically satisfied by solutions of the form

$$\delta\psi(x, z, t) = \psi_0 \sin(\frac{2\pi x}{\lambda}) \cos(\frac{\pi z}{b}) \exp(\alpha t) \tag{17}$$

$$\delta T(x, z, t) = T_0 \cos(\frac{2\pi x}{\lambda}) \cos(\frac{\pi z}{b}) \exp(\alpha t) \tag{18}$$

Putting this ansatz into equations (11) and (12), we require that the coefficient determinant of the resulting linear equation system vanishes for non-trivial solutions δT and $\delta\psi$:

$$\eta \left\{ (\frac{2\pi}{\lambda})^4 + 2\,(\frac{2\pi}{\lambda})^2 (\frac{\pi}{b})^2 + (\frac{\pi}{b})^4 \right\} \cdot \left\{ \alpha + \kappa \left[(\frac{2\pi}{\lambda})^2 + (\frac{\pi}{b})^2 \right] \right\}$$
$$- \rho_0 g \alpha\,(\frac{2\pi}{\lambda})^2 (T_1 - T_0)\frac{1}{b} = 0 \tag{19}$$

Rearranging terms, we get from (19)

$$\alpha = \frac{\kappa}{b^2} \frac{(\frac{2\pi b}{\lambda})^2}{\left[(\frac{2\pi b}{\lambda})^2 + \pi^2\right]^2} \left\{ \text{Ra} - \text{Ra}_{cr}(\frac{2\pi b}{\lambda}) \right\} \quad , \tag{20}$$

with the Rayleigh number $\text{Ra} = \rho_0 g \alpha \Delta T b^3 / \kappa \eta$ and its "critical" value

$$\text{Ra}_{cr}(\frac{2\pi b}{\lambda}) = \frac{\left[(\frac{2\pi b}{\lambda})^2 + \pi^2\right]^3}{(\frac{2\pi b}{\lambda})^2} \quad . \tag{21}$$

This result is interpreted as follows: For long term development, the sign of α determines the stability of the pertubation amplitudes $\delta\psi$ and δT. If $\alpha > 0$, the pertubations will grow unlimited and the reference conductive solution profile will become unstable. If $\alpha < 0$, any small pertubation will decay again with time, and the conductive temperature profile remains stable (see figure).
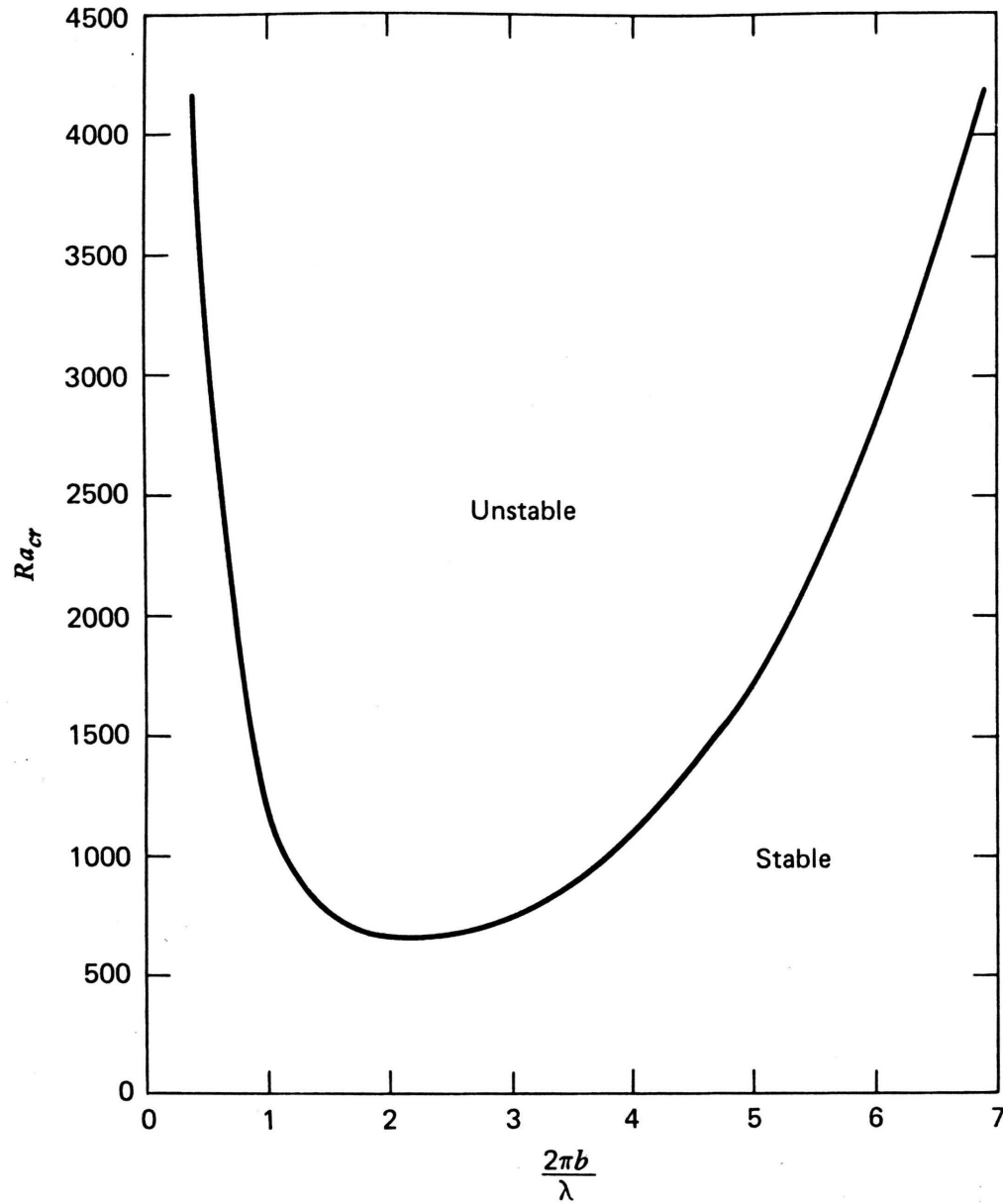
The minimum critical Rayleigh number Ra_{cr}, where the instability is developing first, is obtained be derivating equation (21) with respect to the wavenumber

$$\frac{\partial}{\partial(\frac{2\pi b}{\lambda})} \text{Ra}_{cr} = 0 \quad . \tag{22}$$

This minimum occurs at

$$(\frac{2\pi b}{\lambda}) = \frac{\pi}{\sqrt{2}} \quad ,$$

corresponding to a wavelength of

$$\lambda_{\text{min}} = 2\sqrt{2}b$$

The critical value of Ra at this wavenumber is

$$\text{Ra}_{cr}(\lambda_{\text{min}}) = \frac{27}{4}\pi^4 \sim 657.5 \quad . \tag{23}$$

Appendix (II)

Citcom 2D Isoviscous Convection - Ra = 10^4

Legend:
- 32 x 32 (red)
- 64 x 64 (green)
- 128 x 128 (blue)
- van Keken (magenta)

Axes: Nusselt Number (vertical, 1 to 8), Time (horizontal, 0 to 0.3)

Citcom 2D Isoviscous Convection - Ra = 10^5

Citcom 2D Isoviscous Convection - Ra = 10^6