Dynamics of melt segregation and extraction: from plumes to continental rifting and mid-oceanic spreading

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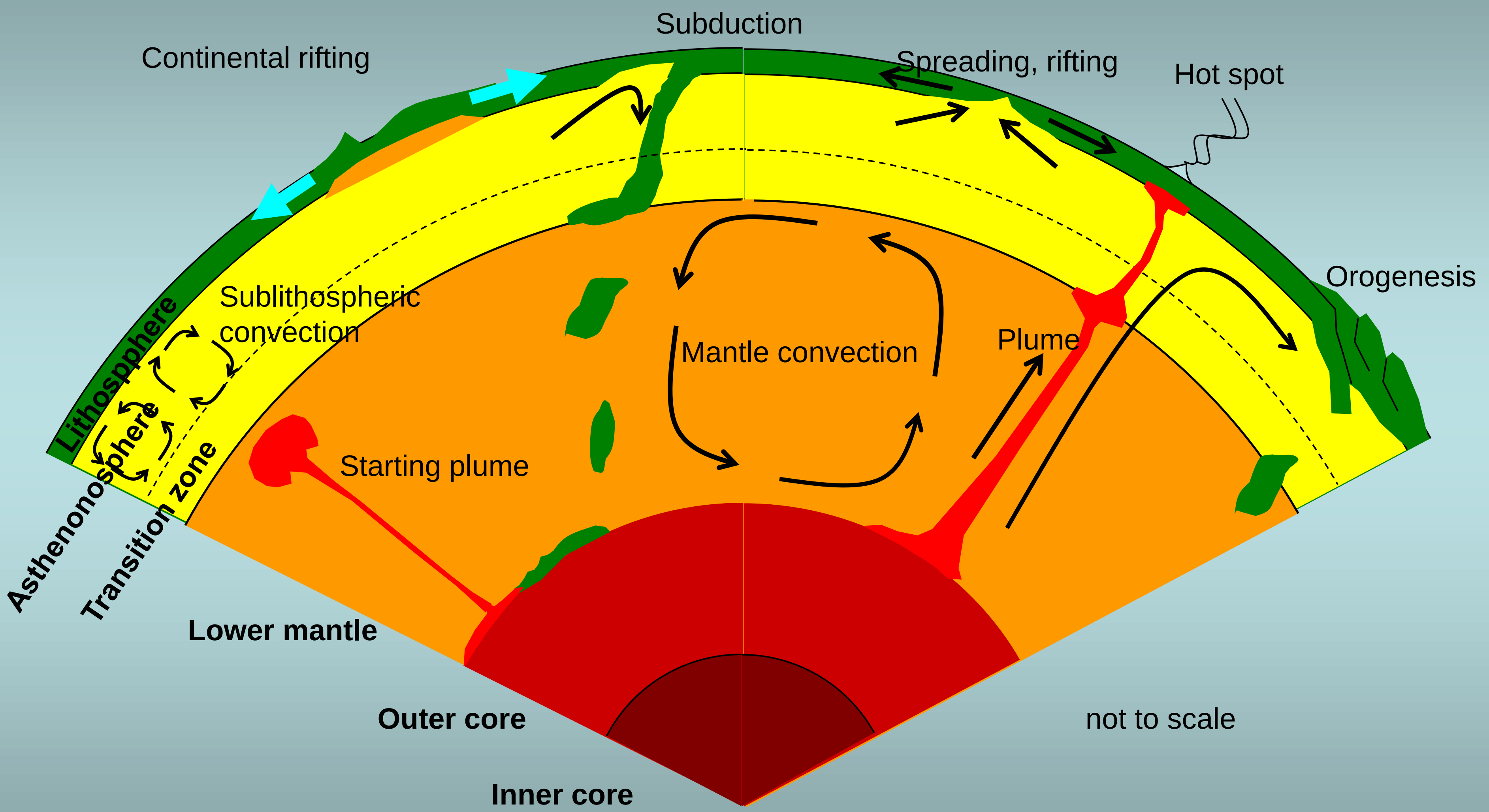
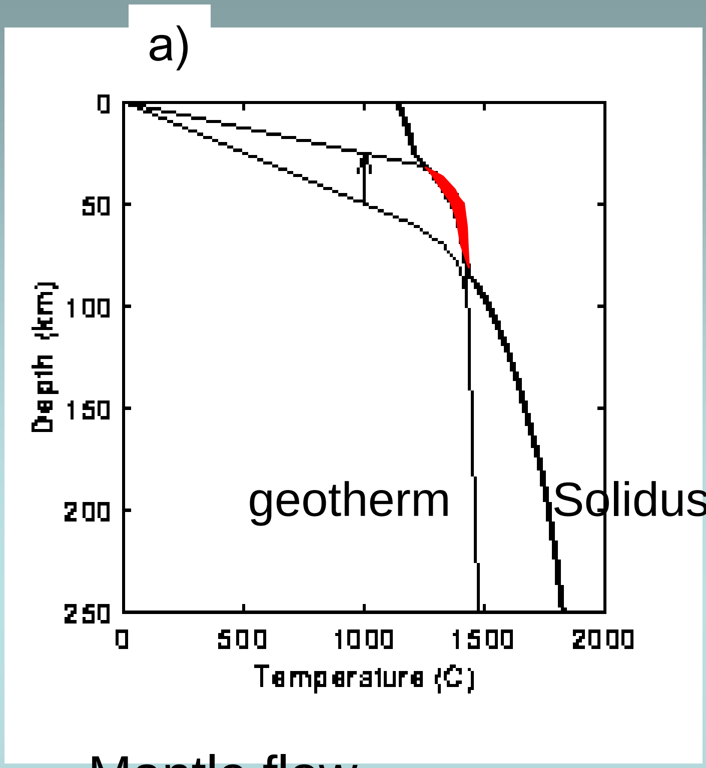
Outline

- Mantle melting zones
- Physics of partially molten source region:
  - Two-phase flow, governing equations
  - melt-porosity dependent shear and bulk viscosity
  - Solution strategies (CBA vs. full compaction)
- Mantle flow with melt percolation → melt accumulation
- Extraction mechanisms
  - Mid ocean ridges, focussing
  - Channeling instability → effective permeability, dykes
  - Melt infiltration into lithospheric base
  - Lithospheric weakening melt extraction/intrusion
- Conclusions

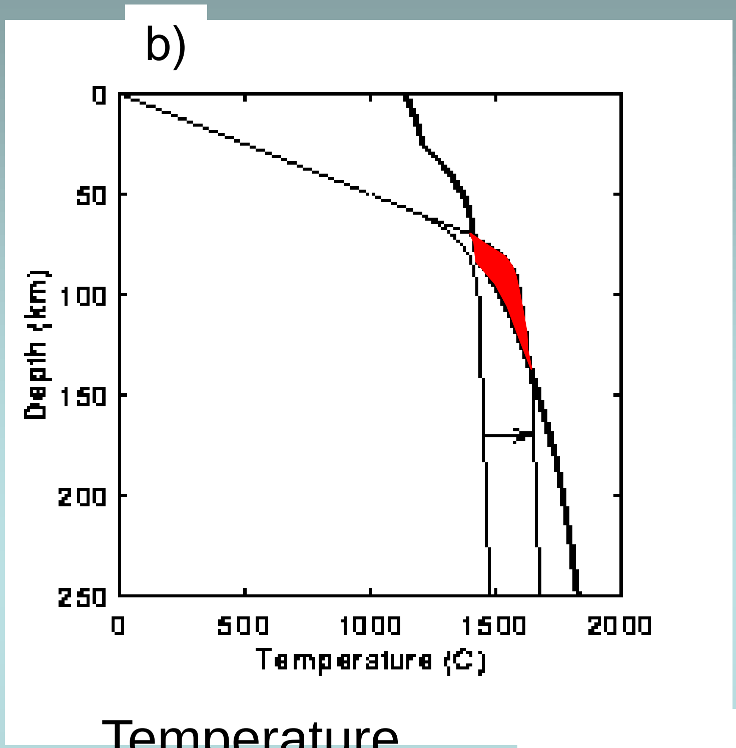
Dynamic earth



Causes of mantle melting

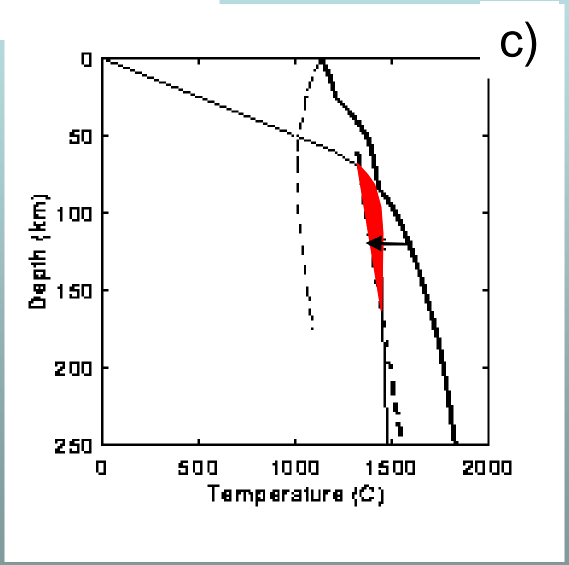


Mantle flow, decompressional melting (Passive rifting)

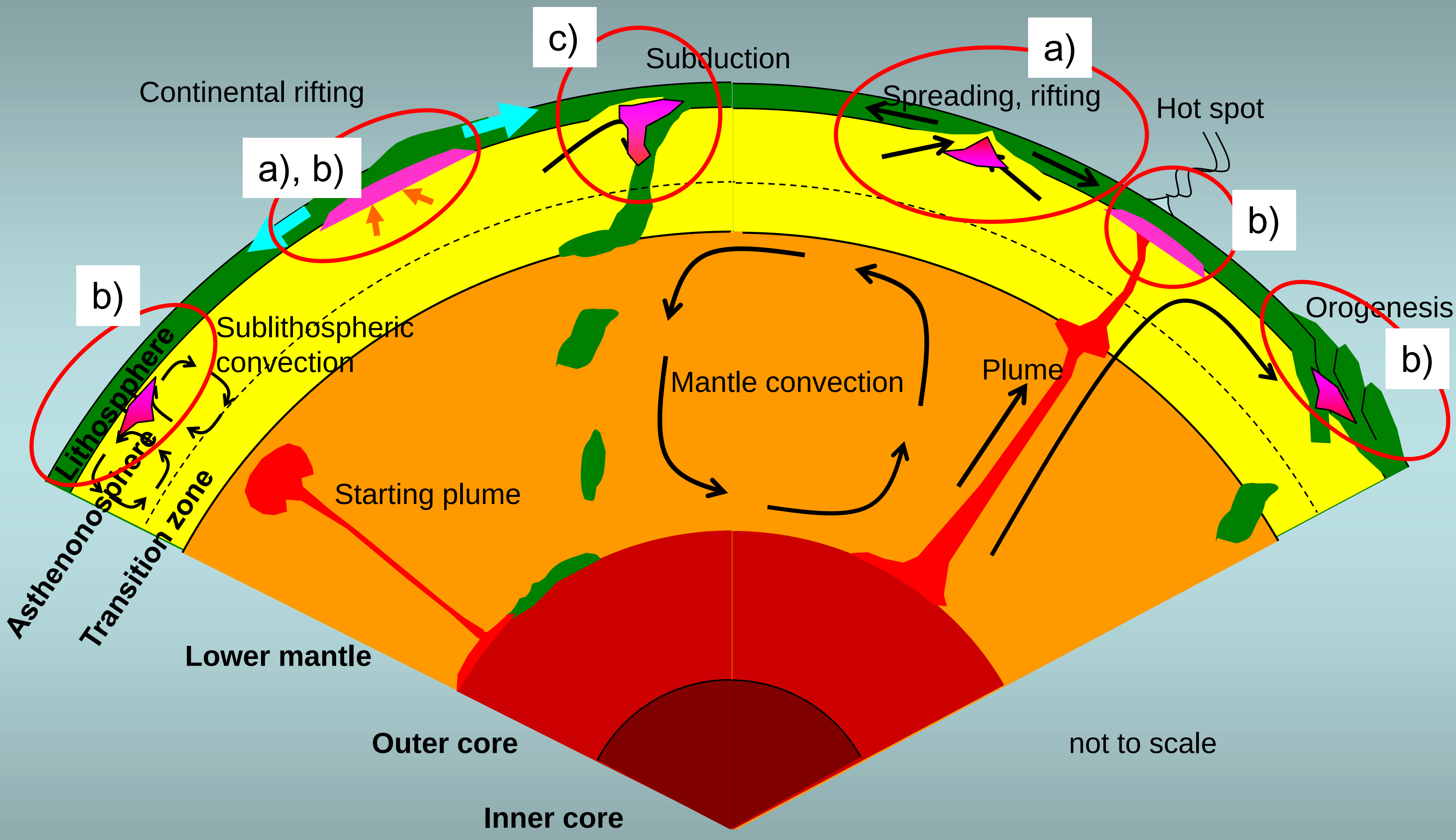


Temperature increase (Active rifting)

Water



The different melt zones



Governing equations of two-phase flow melt - matrix

Three formulations

McKenzie, 1984; …
…Schmeling, 2000; ….

- Melt – matrix: different equations
- No surface tension
- Effective shear and bulk viscosity of matrix
- Breaks down at high melt fractions: viscous stresses in melt neglected

Bercovici et al., 2001, Bercovici and Ricard, 2003,….

- Melt – matrix: symmetric formulation
- Surface tension possible
- Intrinsic melt and matrix viscosities
- Breaks down at high melt fractions: no effective weakening or disaggregation of matrix

Comparison by G. Richard (2010):

$$\frac{-\eta_f}{k_0\phi^{n-1}}\delta V+(1-\phi)\delta\rho g+\nabla\cdot(1-\phi)\eta'\left[\nabla V_m+\nabla V_m^t\right]+\nabla\left[(1-\phi)\left(\frac{\xi}{\eta'}-\frac{2}{3}\right)\eta'\nabla\cdot V_m\right]=0$$

$$\frac{-\eta_f}{k_0\phi^{n-1}}\delta V+(1-\phi)\delta\rho g+\nabla\cdot(1-\phi)\eta_m\left[\nabla V_m+\nabla V_m^t\right]+\nabla\left[(1-\phi)\left(\frac{K}{\phi}-\frac{2}{3}\right)\eta_m\nabla\cdot V_m\right]=0$$

Simpson et al, 2010a,b: Homogenization theory with multiple scale expansion
- Effective bulk and anisotropic shear viscosities
- McKenzie and Bercovici formulations as special cases

Governing equations of two-phase flow melt - matrix

Mass:

$$Melt: \quad \frac{\partial(\rho_f \varphi)}{\partial t} + \vec{\nabla}\cdot(\rho_f\ \varphi \vec{v}_f) = \frac{DM}{Dt}$$

$$Matrix: \quad \frac{\partial \rho_s (1-\varphi)}{\partial t} + \vec{\nabla}\cdot(\rho_s (1-\varphi)\vec{v}_s) = -\frac{DM}{Dt}$$

Momentum:

$$Melt: \quad \vec{v}_f - \vec{v}_s = -\frac{k_\varphi}{\eta_f \varphi}(\vec{\nabla}P + \rho_f\, g\, \delta_{i3})$$

$$Matrix: \quad -\rho g\, \delta_{i3} - \vec{\nabla}P + \frac{\partial \tau_{ij}}{\partial x_j} = 0$$

Rheologic equ of state, permeability-porosity relation (P-T-stress-dependent rheolog

$$\tau_{ij} = \eta_s \left(\frac{\partial v_{si}}{\partial x_j} + \frac{\partial v_{sj}}{\partial x_i} \right) + \delta_{ij}(\eta_b - \frac{2}{3}\eta_s)\vec{\nabla}\cdot\vec{v}_s \qquad k_\varphi = \frac{a^2}{b}\varphi^n$$

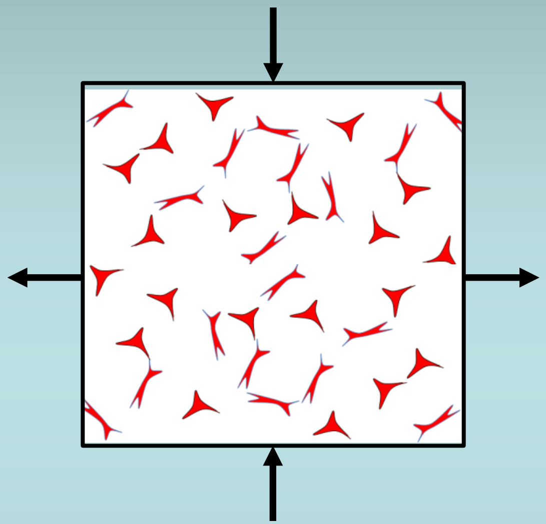
Energy:

$$\rho c_p \left(\frac{\partial T}{\partial t} + \vec{v}\cdot\vec{\nabla}T + \frac{\alpha g}{c_p} v_z T \right) = \vec{\nabla}\cdot(k\vec{\nabla}T) + \rho H + \psi - L\left(\frac{\partial M}{\partial t} + \vec{v}_s \cdot \vec{\nabla}M \right)$$

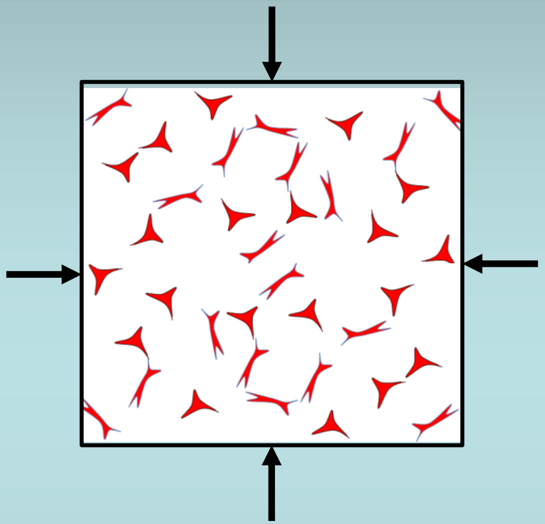
$$\vec{v} = (1-\varphi)\vec{v}_s + \varphi\, \vec{v}_f$$

Effective shear and bulk viscosity of a porous matrix

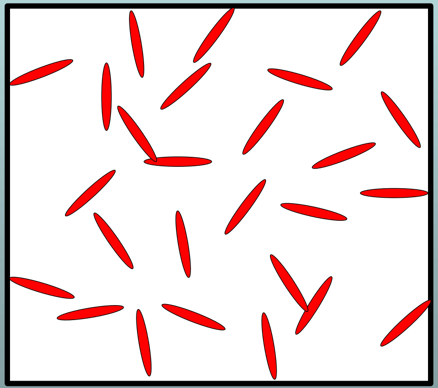
Pure shear → η_s



Compression (empty pores) → η_b

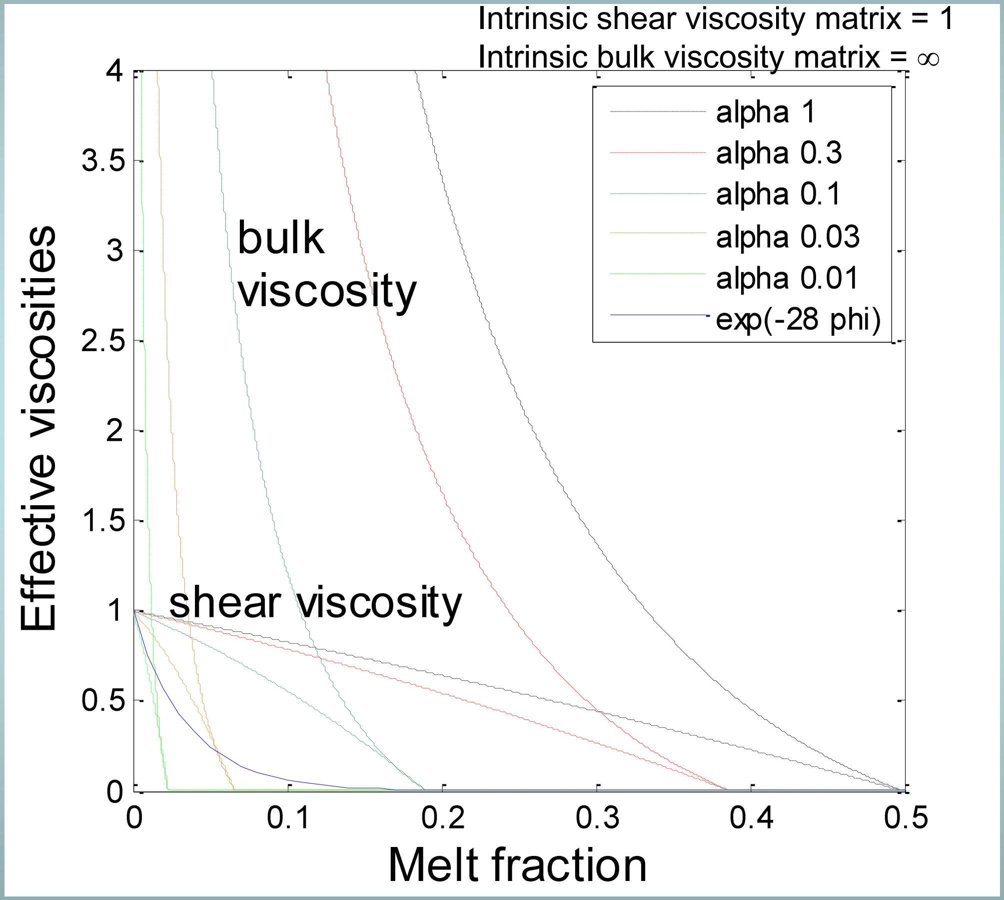


e.g. model with
ellipsoidal inclusions
aspect ratio α



Effective shear and bulk viscosity of a porous matrix

- Ellipsoidal inclusion model: aspect ratio α
  1: spherical melt pockets
  << 1: melt films wetting grain boundaries
  (based on self-consistent elastic moduli formulation, Schmeling (1985), now as Matlab routine, see homepage Schmeling)
- exp(-28 φ): Experimental data of partially molten peridotite (Kohlstedt)
- Predicts disaggregation of matrix at φ_{cr} → two-phase flow formulation breaks down
- Low shear & bulk viscosities: → numerical instabilities
- High bulk viscosities: slow convergence



Approximative formulas: $\eta_{eff} = \eta_0 \left(1 - \varphi \frac{1+c_2\alpha^{1.25}}{c_1\alpha}\right), \eta_{effbulk} = \eta_0 c_{b1} \frac{(c_{b2}-\varphi)}{\varphi}$

Solution strategies of melt – matrix equations in mantle convection scenarios

Zero order approximation: Determine amount of melting by supersolidus T,
No melt flow equation, no compaction

The **Compaction Boussinesq Approximation CBA**

$$Matrix: \quad -\rho g\,\delta_{i3} - \vec{\nabla} P + \frac{\partial \tau_{ij}}{\partial x_j} = 0$$

(Schmeling, 2000)

$$\tau_{ij} = \eta_s\left(\frac{\partial v_{si}}{\partial x_j} + \frac{\partial v_{sj}}{\partial x_i}\right) + \cancel{\delta_{ij}\left(\eta_b - \frac{2}{3}\eta_s\right)\vec{\nabla}\vec{v}_s}$$

Assume div v_s = 0 everywhere except in the melt flow equation

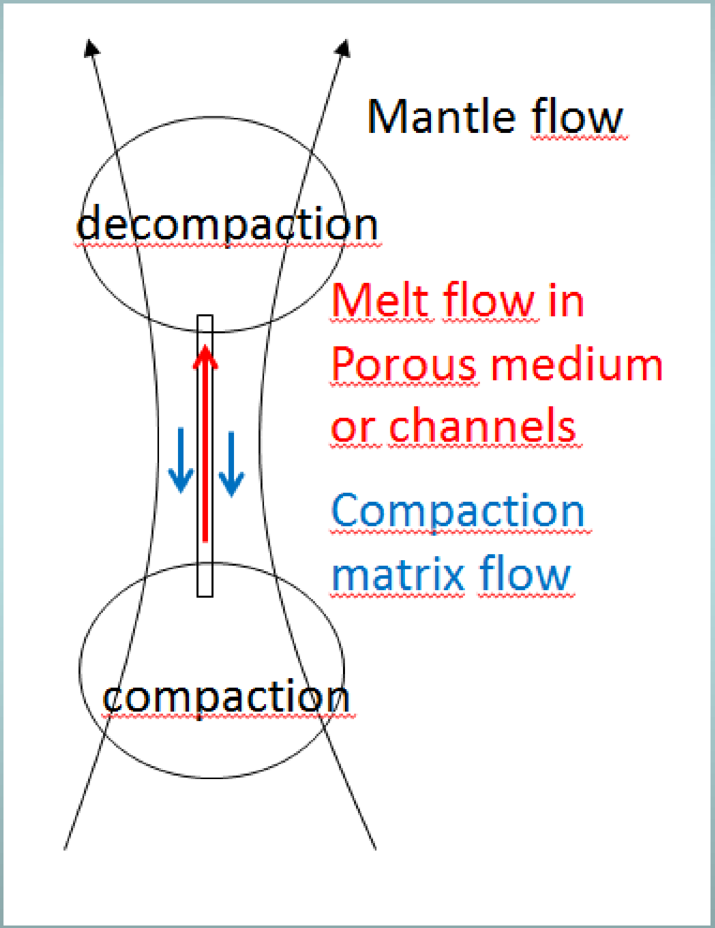
$$\vec{v}_f - \vec{v}_s = \frac{1}{Rt}\varphi^{n-1}\left(Rm(1-\varphi)\delta_{i3} - \frac{\partial}{\partial x_j}\left(\eta_s\left(\frac{\partial v_{si}}{\partial x_j} + \frac{\partial v_{sj}}{\partial x_i}\right)\right) - \nabla\left(\left(\eta_b - \frac{2}{3}\eta_s\right)\nabla\cdot\vec{v}_s\right)\right)$$

where div v is derived from the mass conservation equations

In 2D → Stream function formulation

$$\left(\frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial x^2}\right)\left[\eta\left(\frac{\partial^2\psi}{\partial z^2} - \frac{\partial^2\psi}{\partial x^2}\right)\right] + 4\frac{\partial^2}{\partial x\partial z}\left(\eta\frac{\partial^2\psi}{\partial x\partial z}\right) = Ra\frac{\partial T}{\partial x}$$

$$\nabla^4\psi = Ra\frac{\partial T}{\partial x}$$



→ Melt „feels“ compaction pressure, matrix moves as incompressible fluid

Solution strategies of melt – matrix equations in mantle convection scenarios

Dropping the CBA (following Sramek et al. 2007)

Decompose matrix velocity into incompressible and irrotational (compaction) flow:

$$\vec{v}_s = \vec{v}_1 + \vec{v}_2 = \begin{pmatrix} -\frac{\partial \psi}{\partial z} \\ \frac{\partial \psi}{\partial x} \end{pmatrix} + \begin{pmatrix} \frac{\partial \chi}{\partial x} \\ \frac{\partial \chi}{\partial z} \end{pmatrix}$$

ψ - stream function, χ - irrotational velocity potential: $\nabla \cdot \vec{v}_s = \nabla^2 \chi$

→ Matrix momentum equation

$$\left(\frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial x^2}\right)\left[\eta_s\left(\frac{\partial^2 \psi}{\partial z^2} - \frac{\partial^2 \psi}{\partial x^2}\right)\right] + 4\frac{\partial^2}{\partial x \partial z}\left(\eta_s \frac{\partial^2 \psi}{\partial x \partial z}\right) = Ra\,\frac{\partial T}{\partial x} + A(\chi)$$

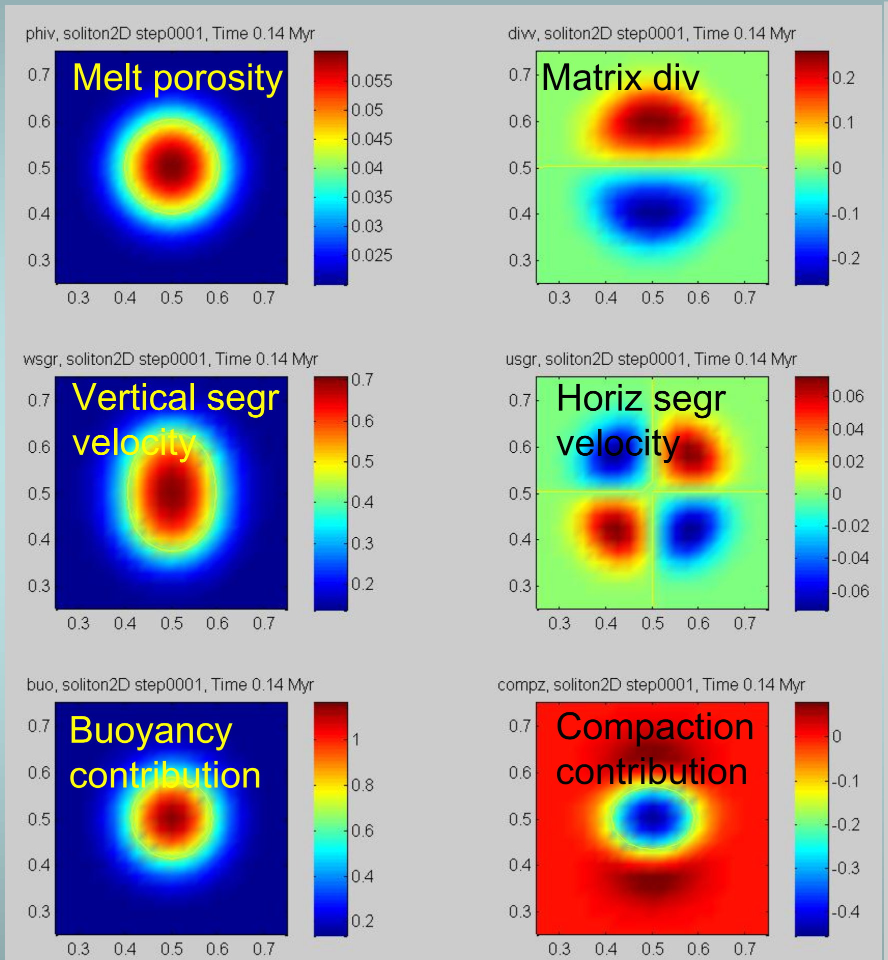
with $A(\chi) = -2\frac{\partial^2}{\partial x \partial z}\left[\eta_s\left(\frac{\partial^2 \chi}{\partial z^2} - \frac{\partial^2 \chi}{\partial x^2}\right)\right] + 2\left(\frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial x^2}\right)\left(\eta_s \frac{\partial^2 \chi}{\partial x \partial z}\right)$

- Compaction term adds as a load to the convection equation, χ to be derived from melt mass conservation equation
- $A(\chi) = 0$ for const viscosity
- Matrix momentum equation does not depend explicitly on bulk viscosity

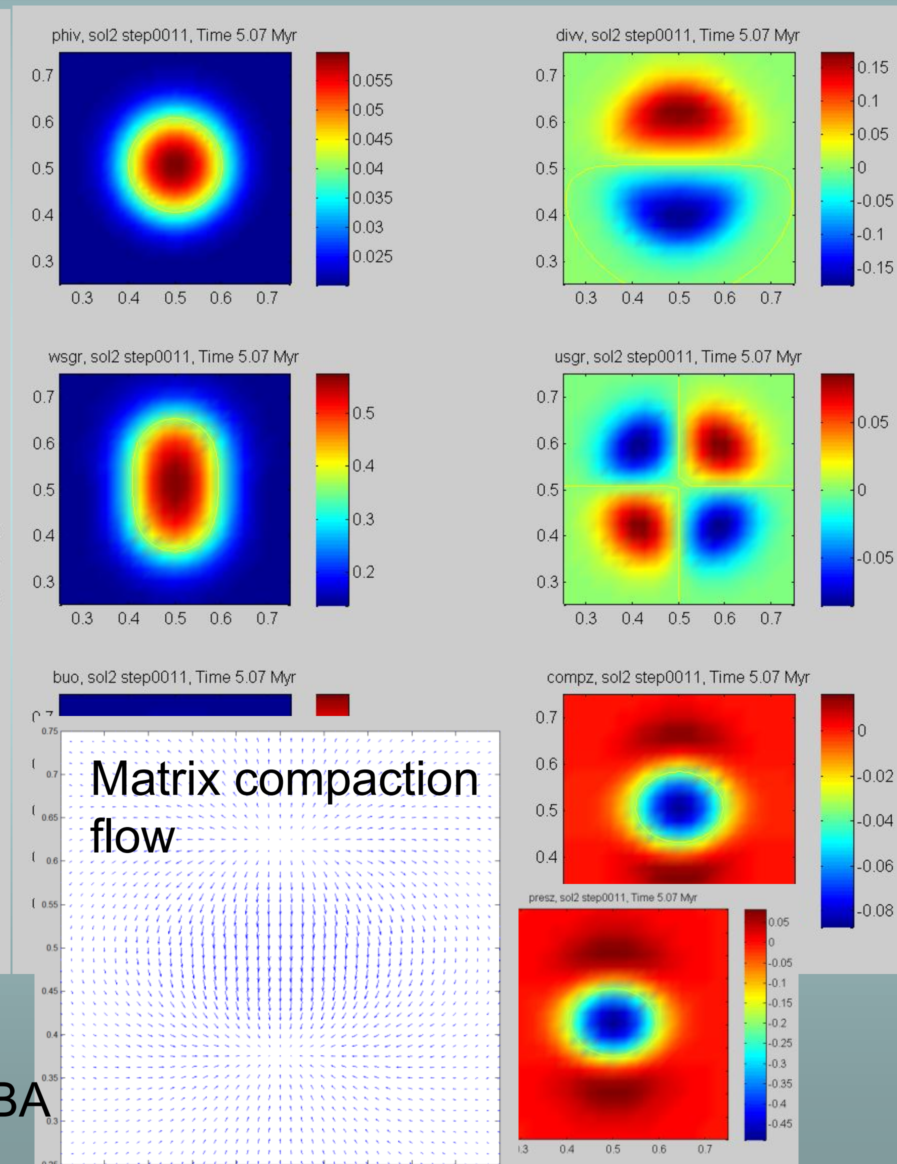
Comparison melt porosity wave

(solitary wave, const viscosity, Barcilon and Lovera, 1989)

With CBA

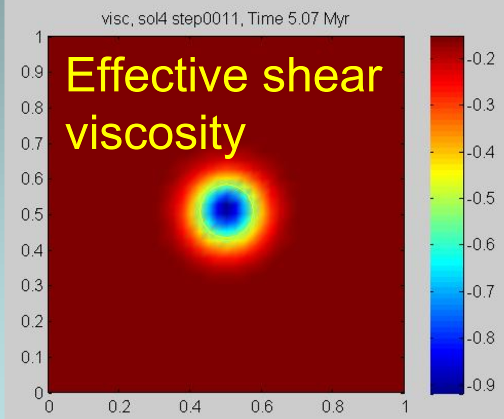


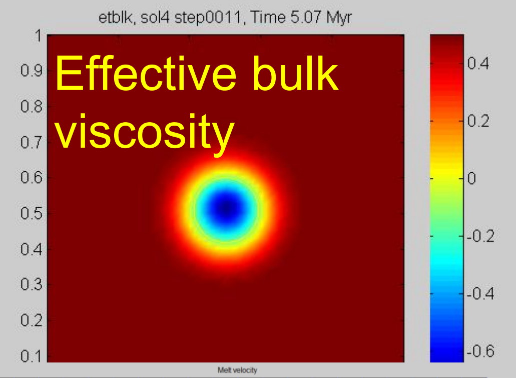
With full compaction



→ Full compaction reduces div v and Segregation somewhat compared to CBA

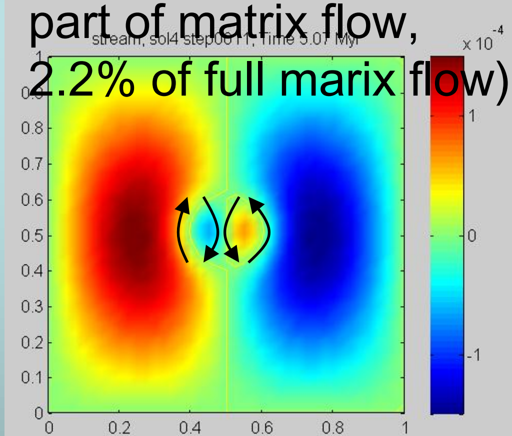
Now with porosity dependent shear and bulk viscosity

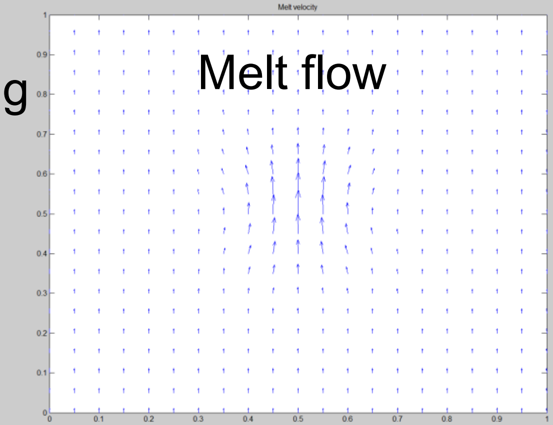


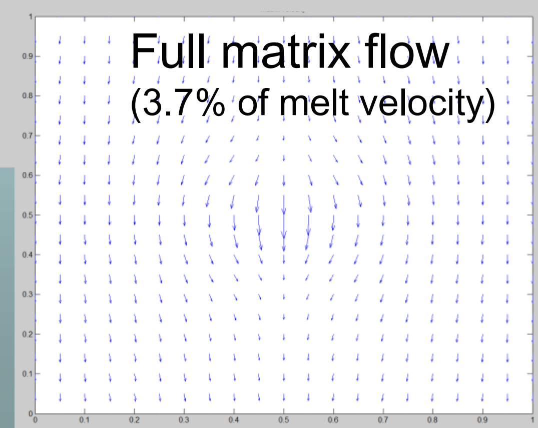


Now stream function (=rotational flow) is influenced by compaction: Low viscosity attracts streamlines and induces (matrix flow) convection cell

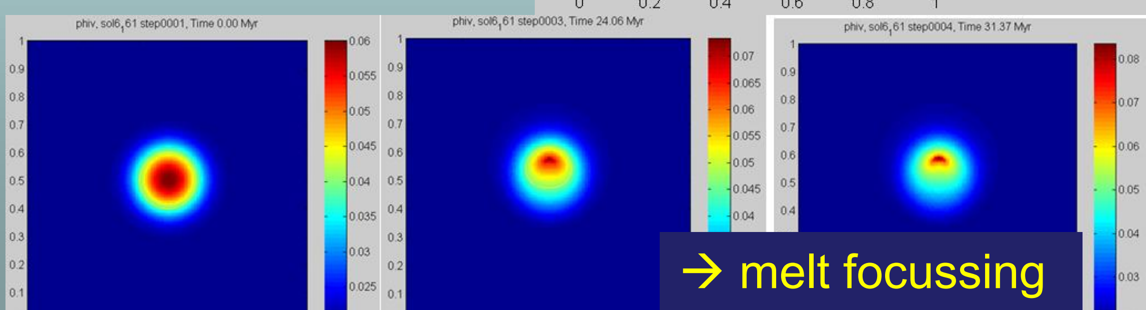
Streamfunction (non-compacting part of matrix flow, 2.2% of full marix flow)







Further time evolution:



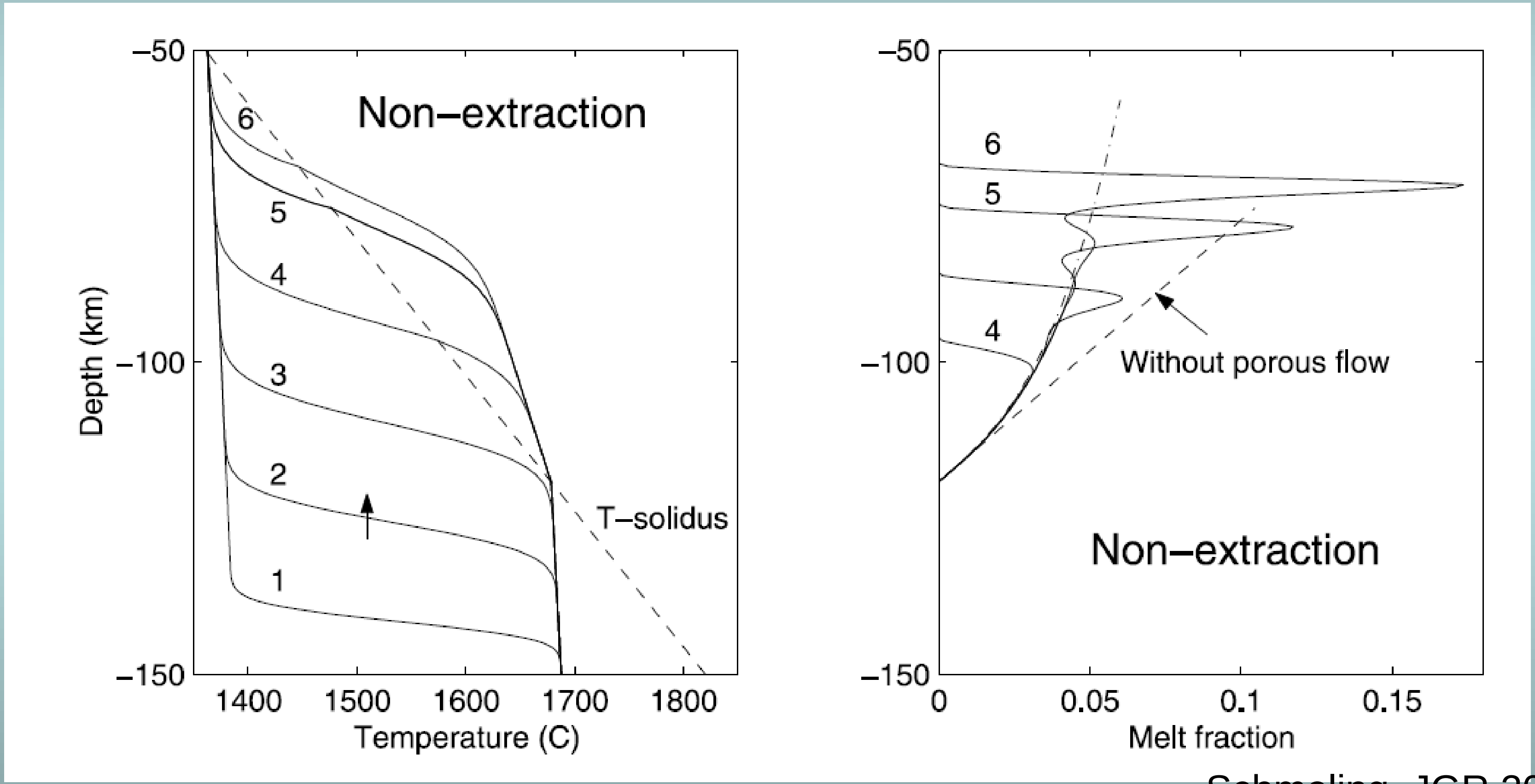
→ melt focussing

Mantle flow with melt percolation

- How to segregate and extract and the melt from plumes and ridges?

1D, rising T-anomaly (plume), const matrix velocity:

- Melt percolation slow
- Only within partially molten (source) zone
- Accumulation near solidus temperature

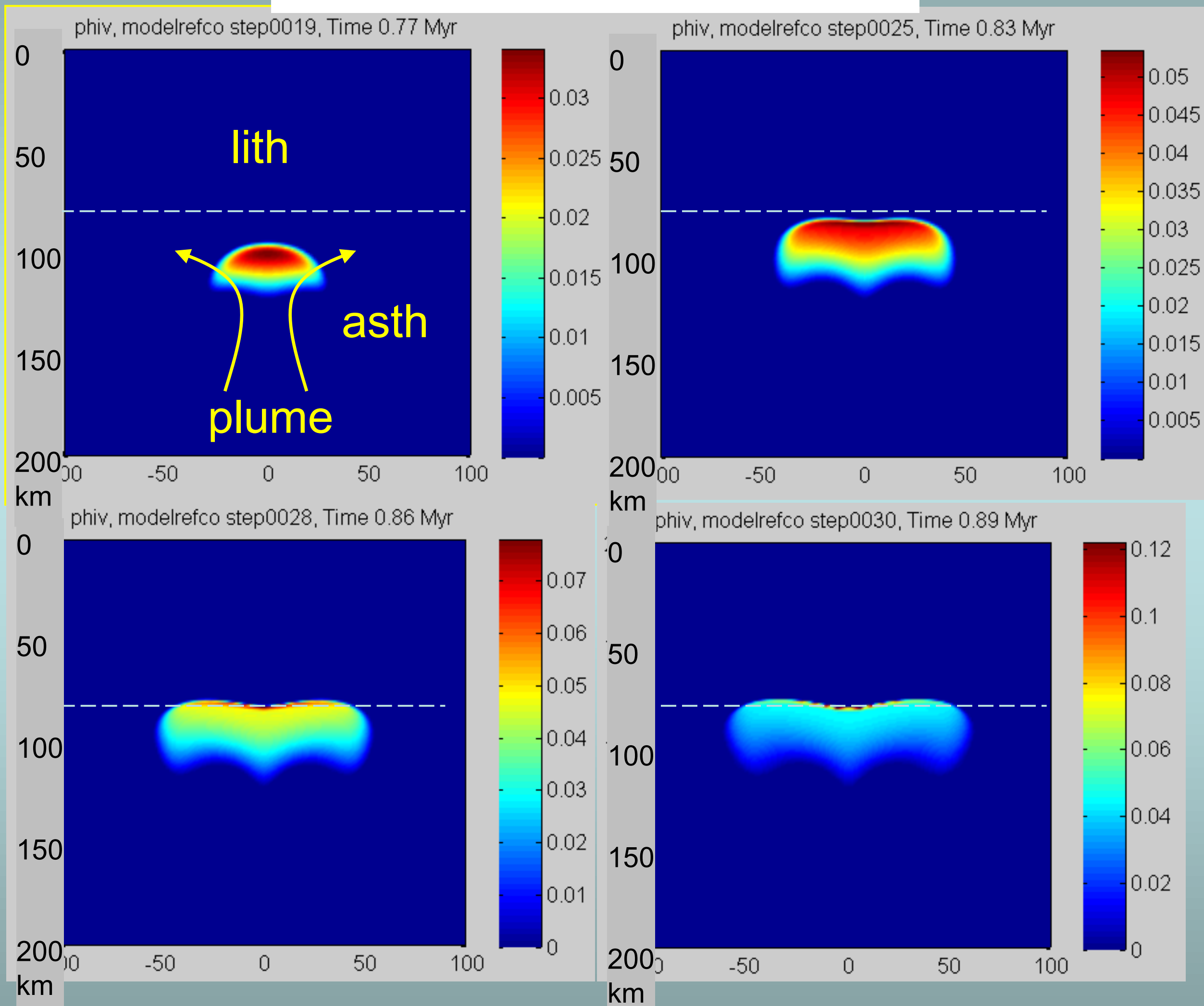


Schmeling, JGR 2006

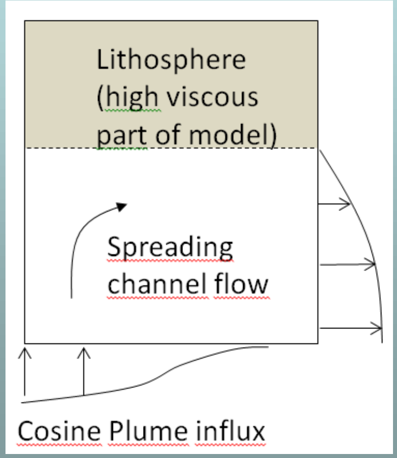
⇒ Fluid-velocities cm/yr to dm/yr

Melt accumulation in an plume head arriving at lithospheric base

Melt fraction at different times

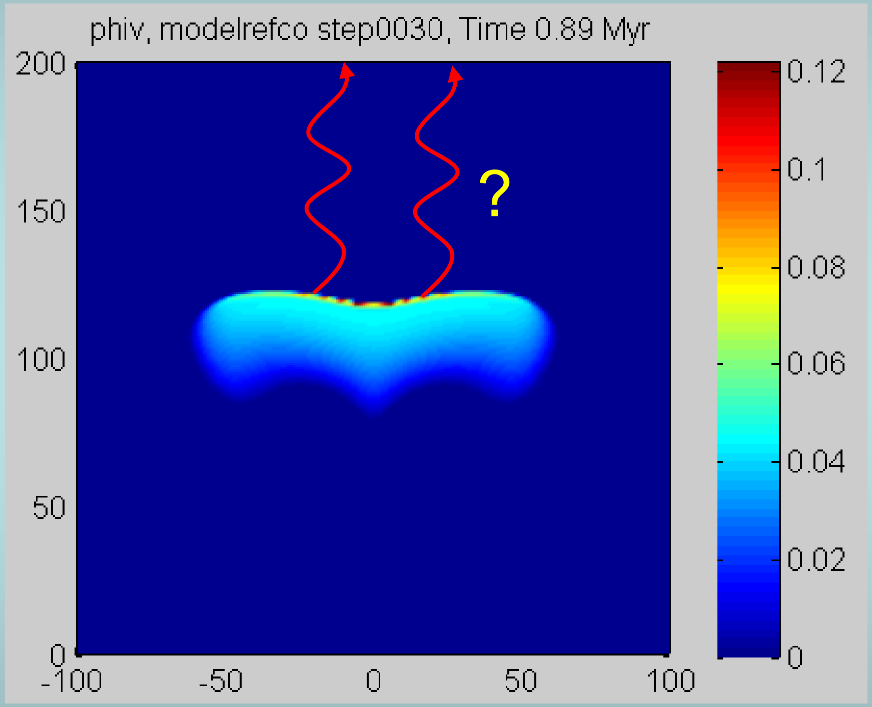


- Full two-phase flow solution with compaction
- Melting-freezing with simplified binary system
- Non-Newtonian P-T-dependent rheology
- Plume influx 10 cm/yr
- Plume excess temp 150K



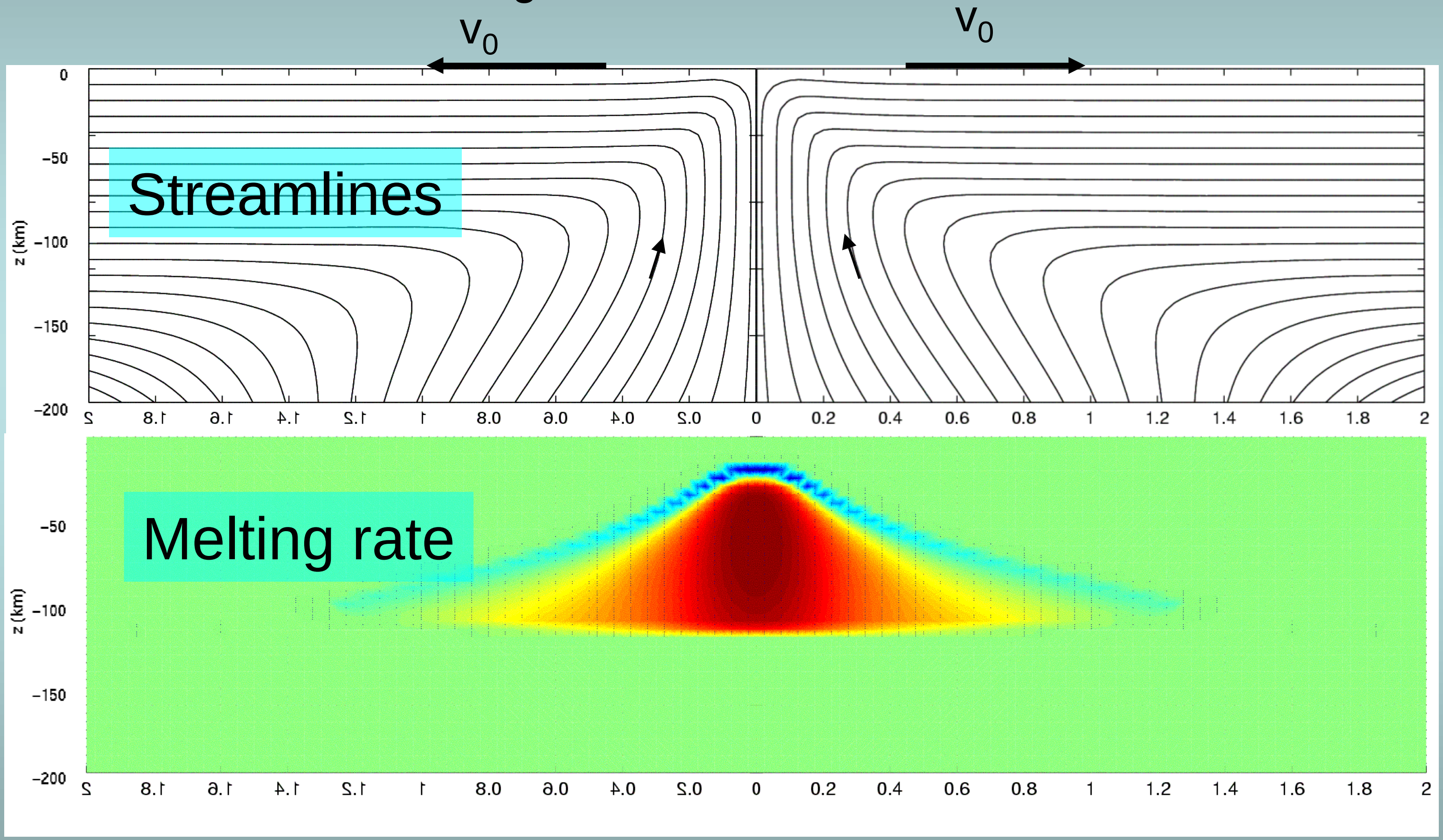
Melt accumulation to > 20%
→ Convergence problems due to sharp contrasts

How does melt ascend from source region to surface?

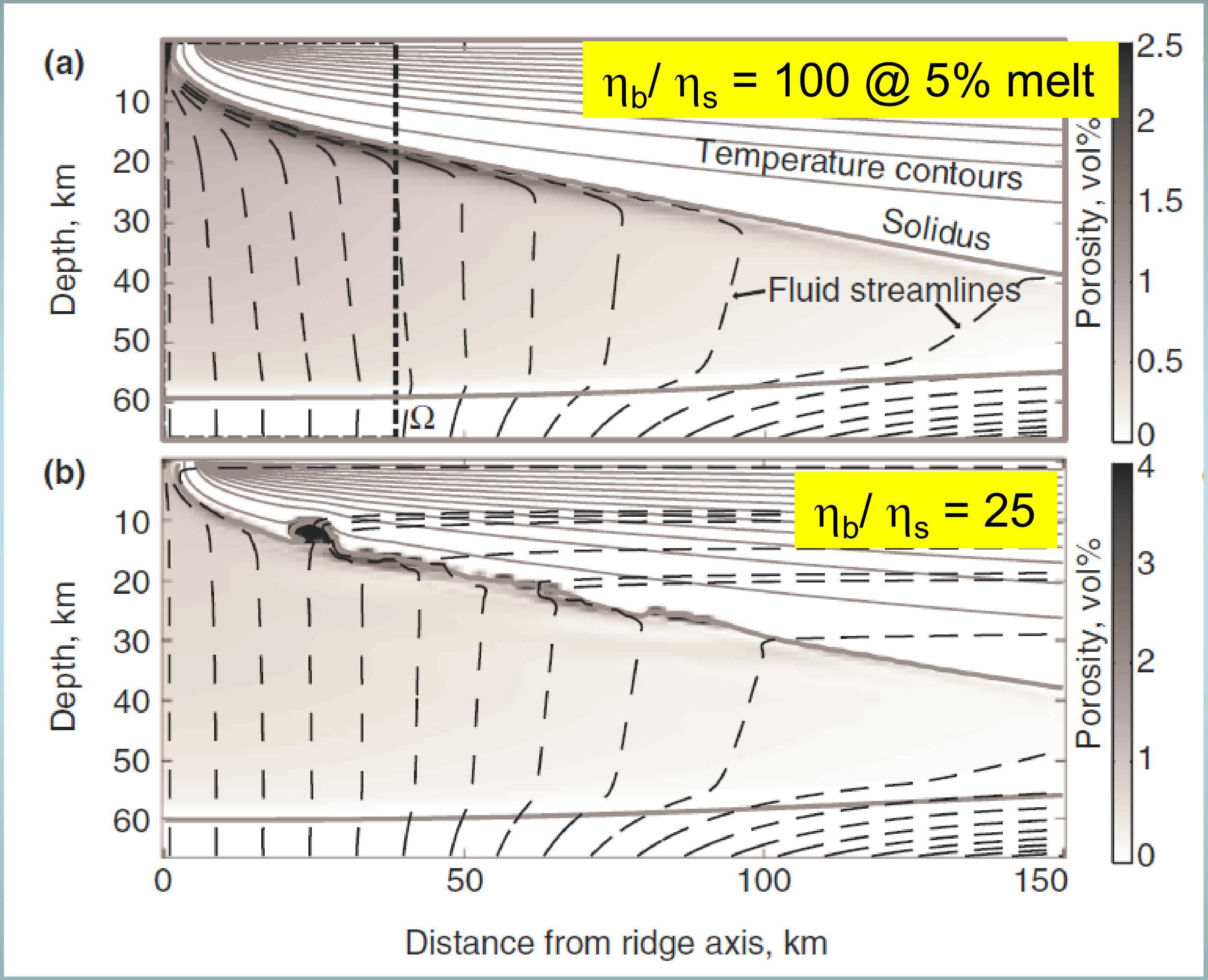


1.) Mid-ocean ridges
2.) Sublith. convection / plumes, cont rifting

Width of crust accretion zone O(10 km)
Width of melting zone O(100km)
Focussing? Crustal thickness of 6km?



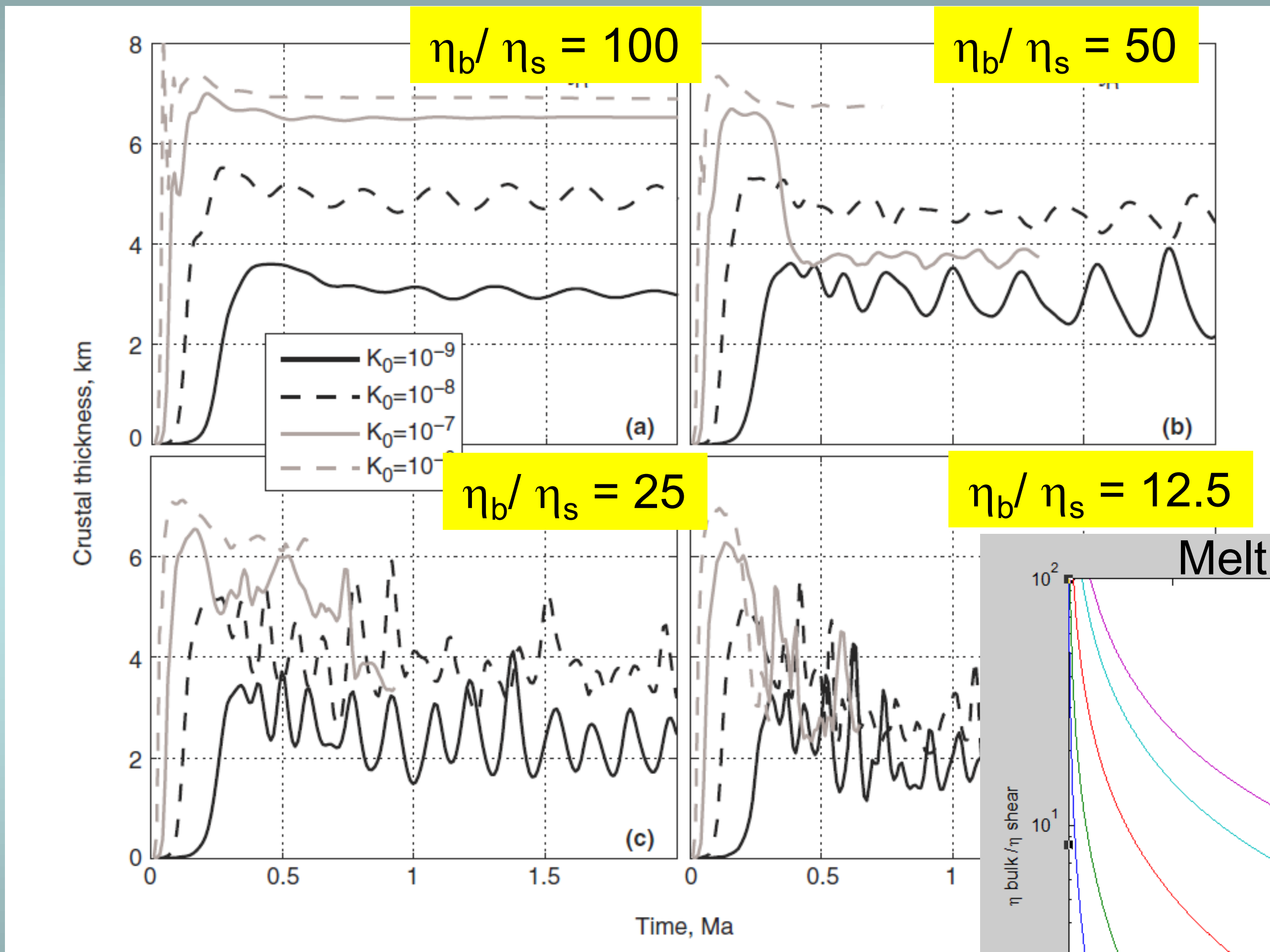
Katz 2008: **Importance of bulk viscosity**



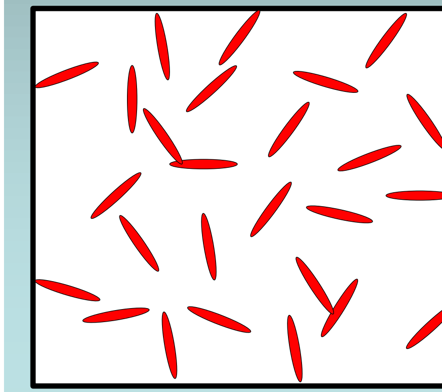
- Solving the full equations (enthalphy formulation)
- v_0 = 5 cm/a
- η_s = const
- 6 km wide accretion zone ($\frac{\partial H}{\partial z} = 0$)
- $k_0 = 10^{-7}$ m^2 where $k = k_o \varphi^n$

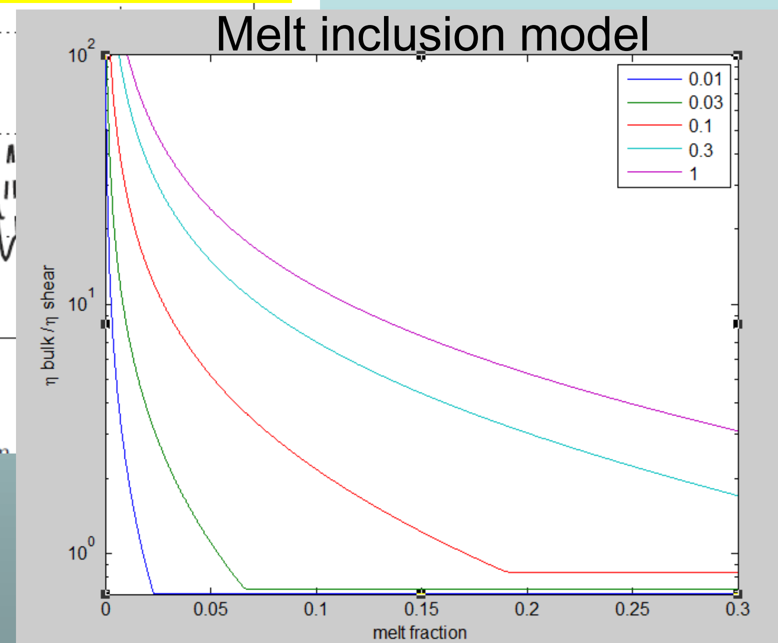
high η_b / η_s :
→ steady state
→ Strong focussing
→ Effective extraction
→ Thick crust (6km)

Katz 2008: Varying bulk viscosity and permeability constant



Crustal thickness as a function of time. Each panel shows the evolution of crustal thickness from





→ Successful selfconsistent melt focussing model

How does melt ascend from source region to surface?

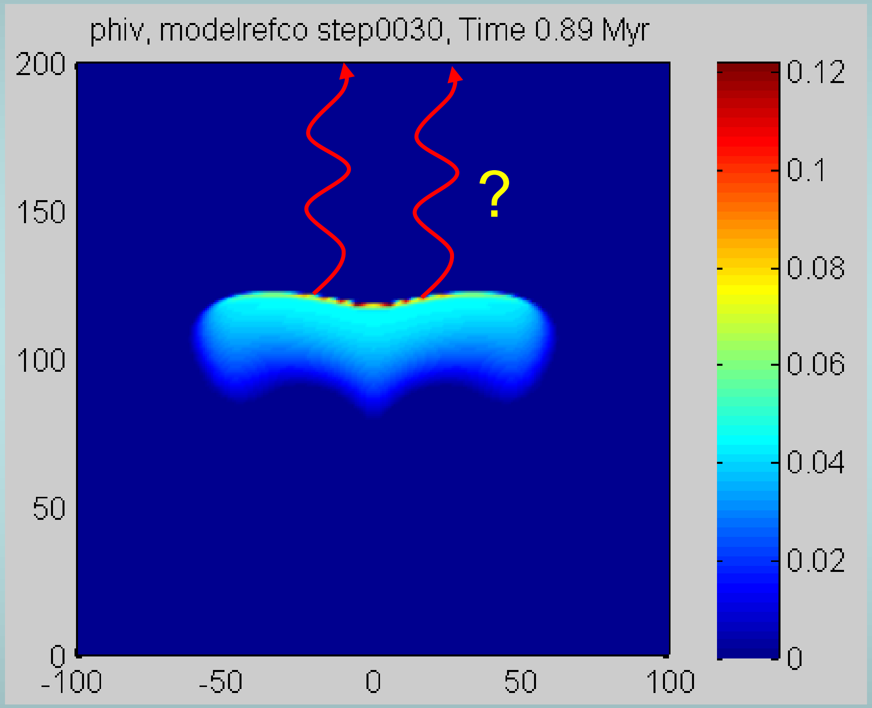
- Dykes

↑

- Channelling instability

↑

- Porous flow



1.) Mid-ocean ridges

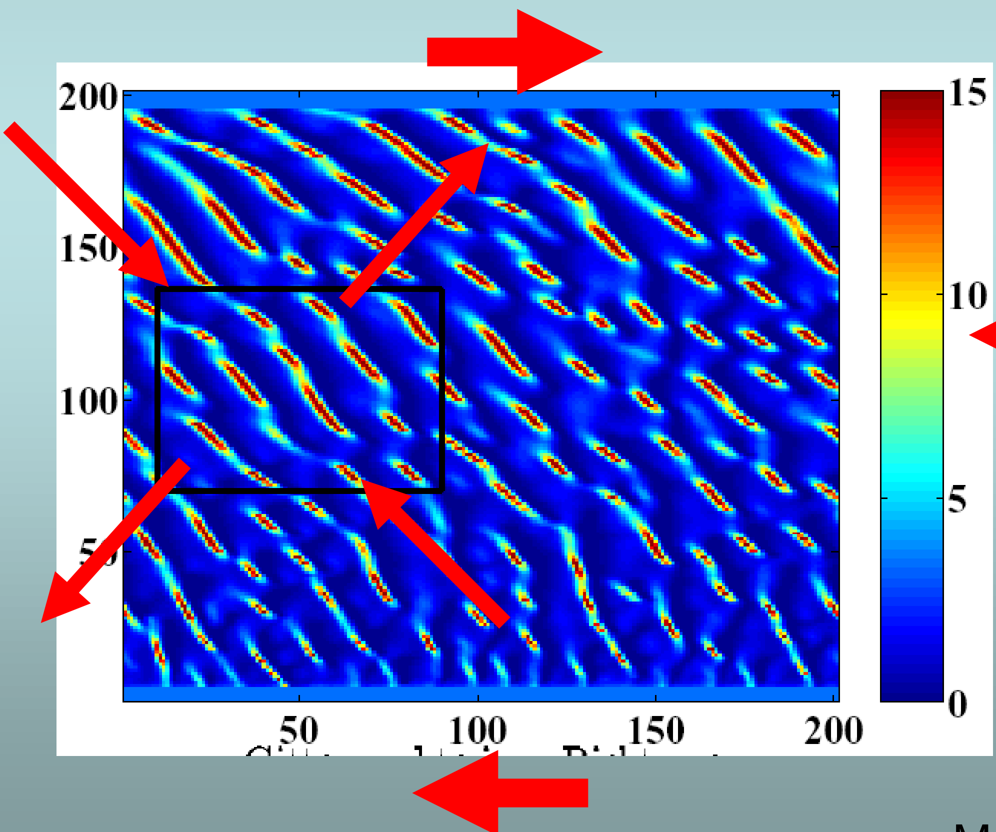
2.) Sublith. convection / plumes, cont rifting

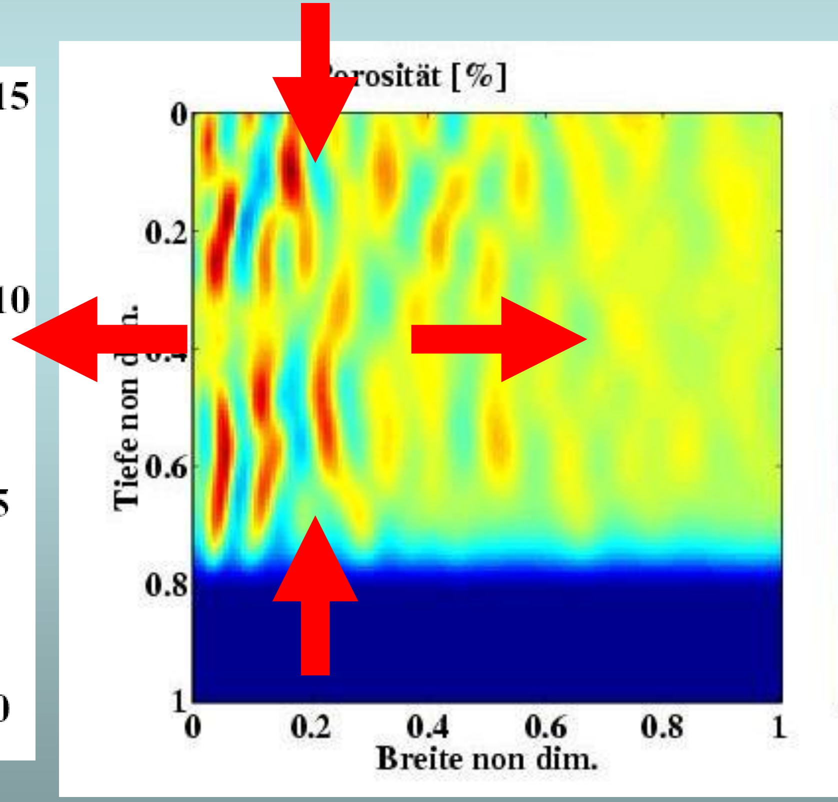
The Channel instability:
Melt ascent through oriented melt channels

Stevenson, 1989,
Richarson, 1998,
Golabek, et al 2008
Müller, Schmeling in rev
Katz et al. 2006

Partially molten rock under deformation:

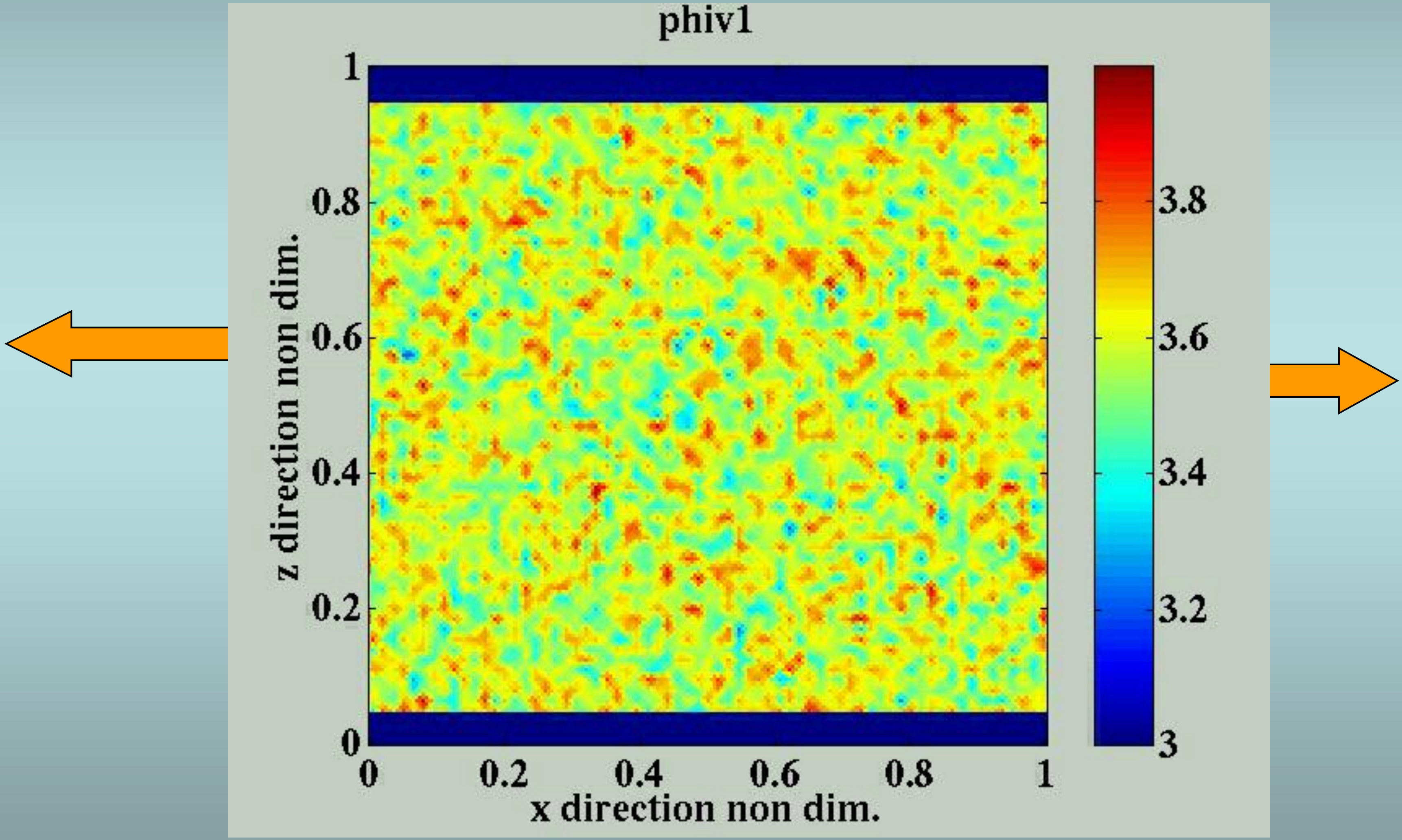
Channelling perpendicular to maximimum tensile stress
(Feed back – porosity – viscosity – pressure gradient)





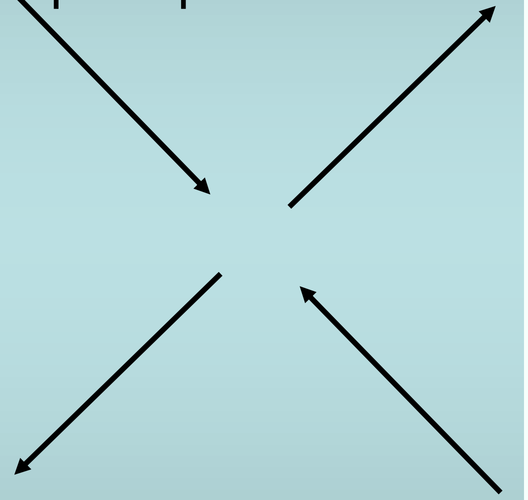
Müller PhD thesis 2008

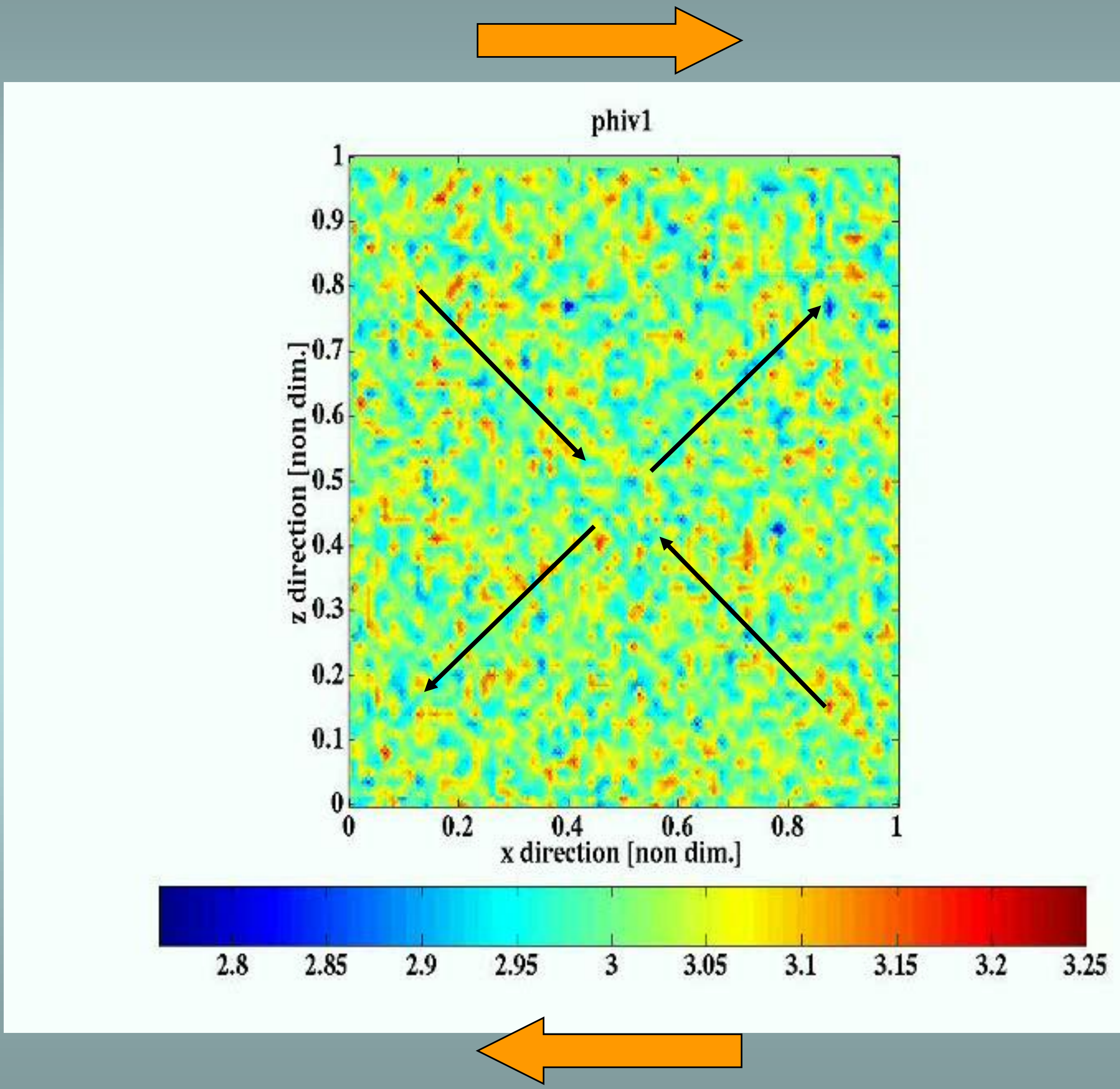
Initial melt distribution:
3% melt with statistical fluctiations (±0.05%)



Simple shear
+ melt buoyancy

principal stress

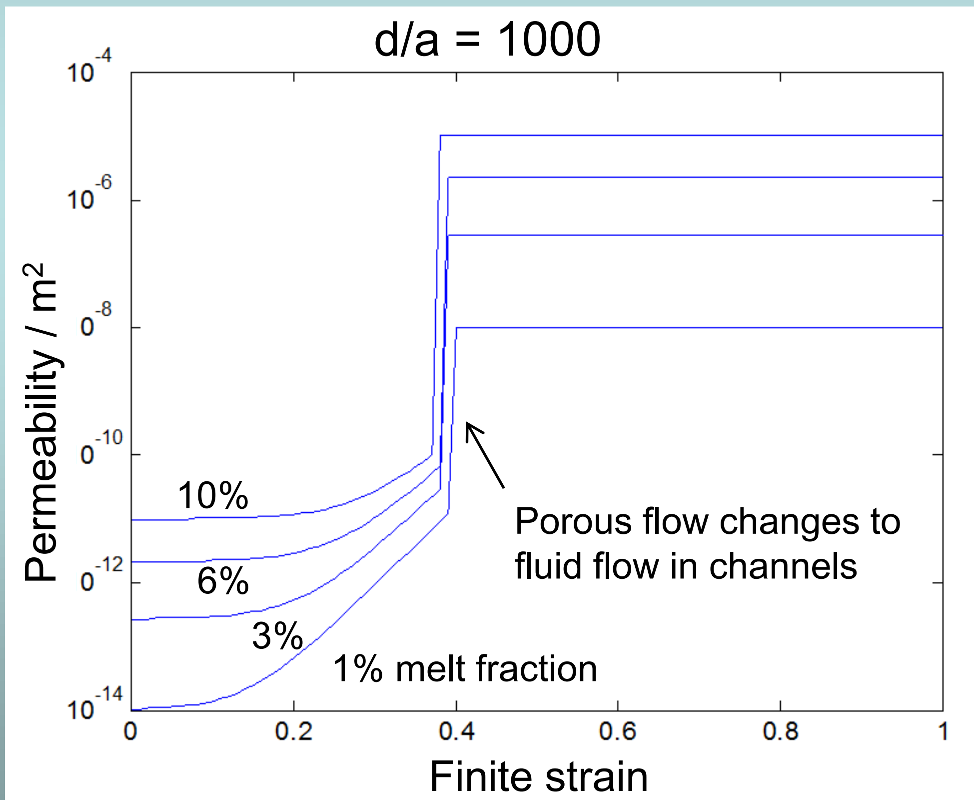


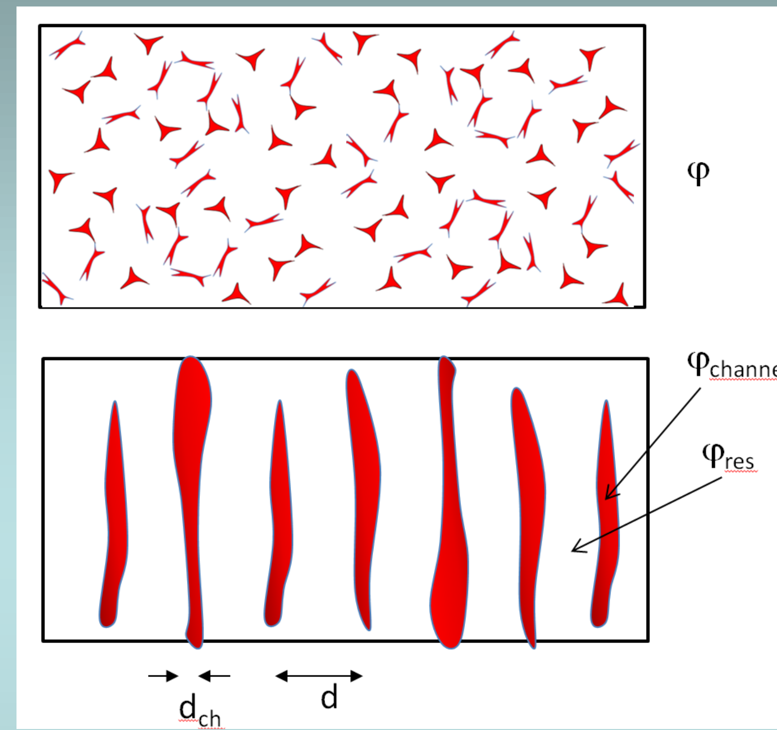


Consequences

1) Strong increase in effective permeability

Construct effective permeability law





$$k_{eff} = \varphi \frac{a^2}{b} \varphi_{channel}{}^{n} + (1-\varphi)\frac{a^2}{b}\varphi_{res}{}^{n}$$

$$k_{effchann} = \frac{d^2}{b}\varphi^{n}$$

$$k_{comb} = \begin{cases} k_{eff}(\gamma_{xy}) & f(\gamma_{xy}) < f_{cr} \\ k_{effchann} & f(\gamma_{xy}) \geq f_{cr} \end{cases}$$

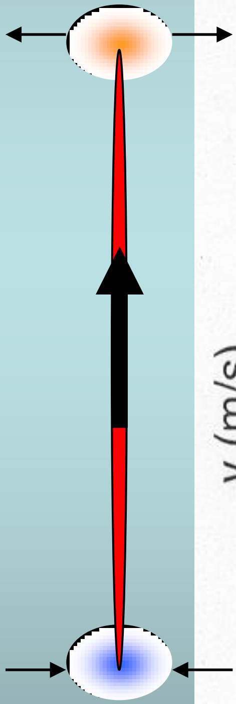
Consequences

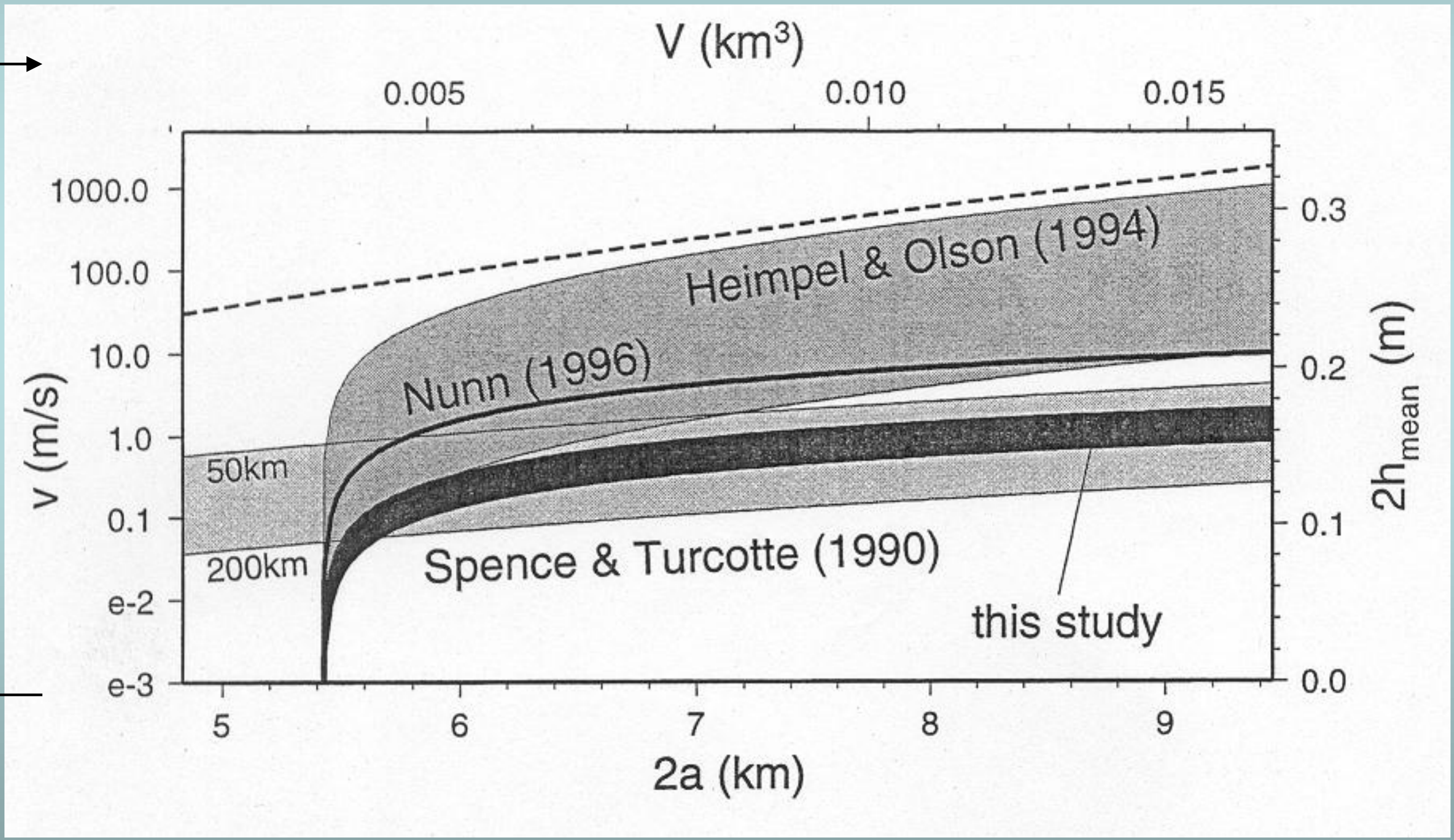
2) Vertical channels, may initiate dyking

Buoyancy driven propagation of magma filled dykes

(crack propagation in elastic media)

(Dahm 2000)



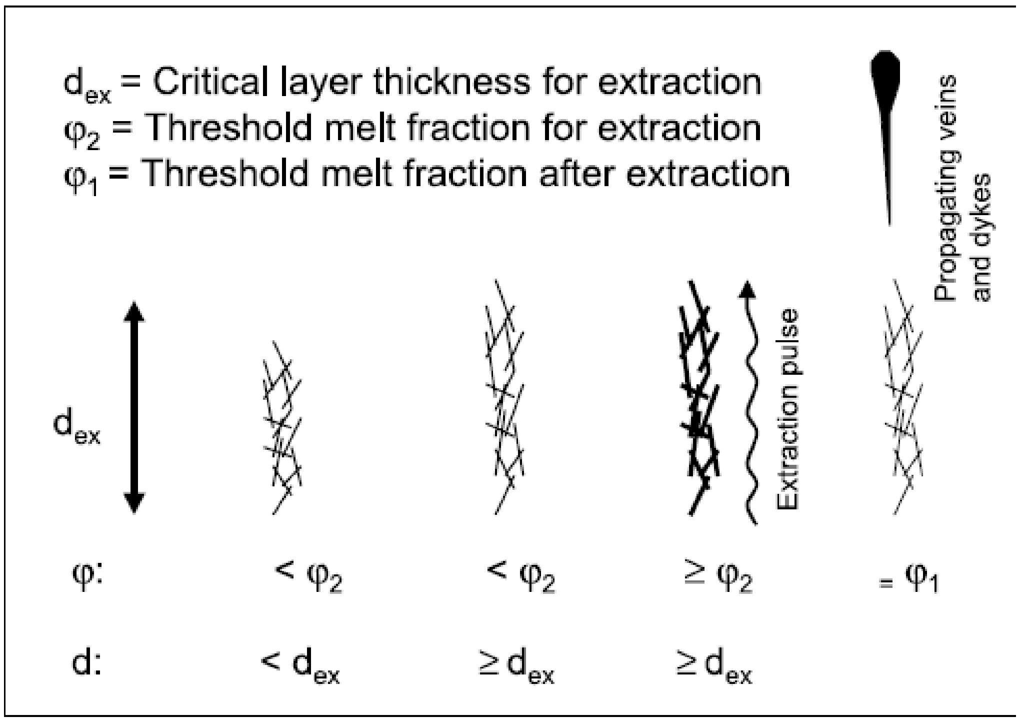


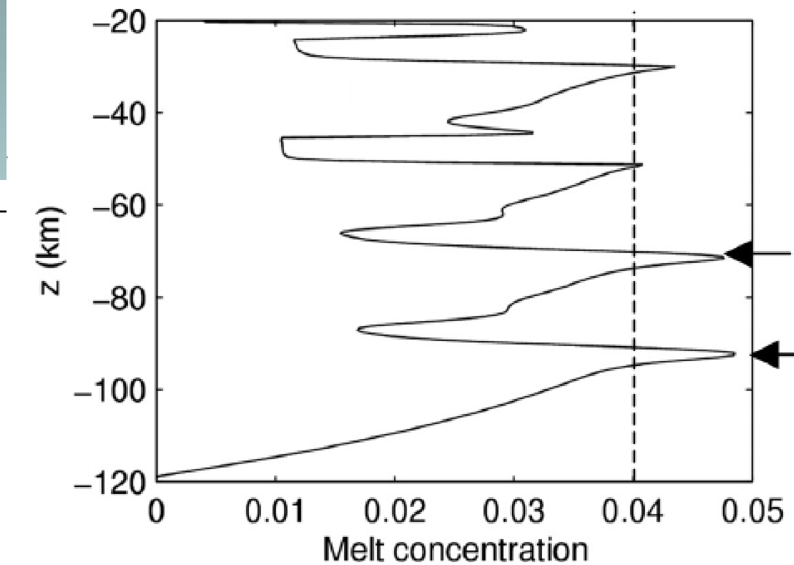
$\rightarrow$ Minimum length required, e.g. resulting from melt channels

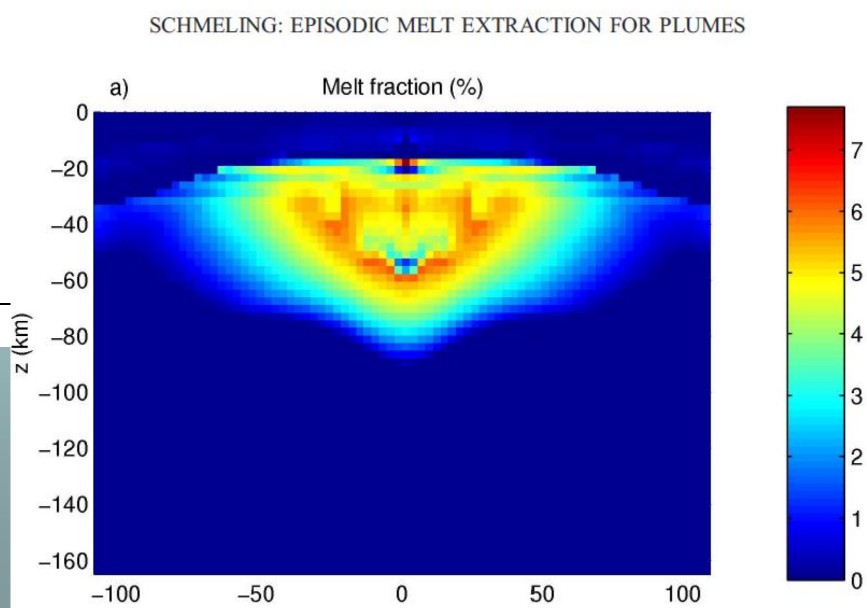
→ Simple melt extraction model

Schmeling 2006

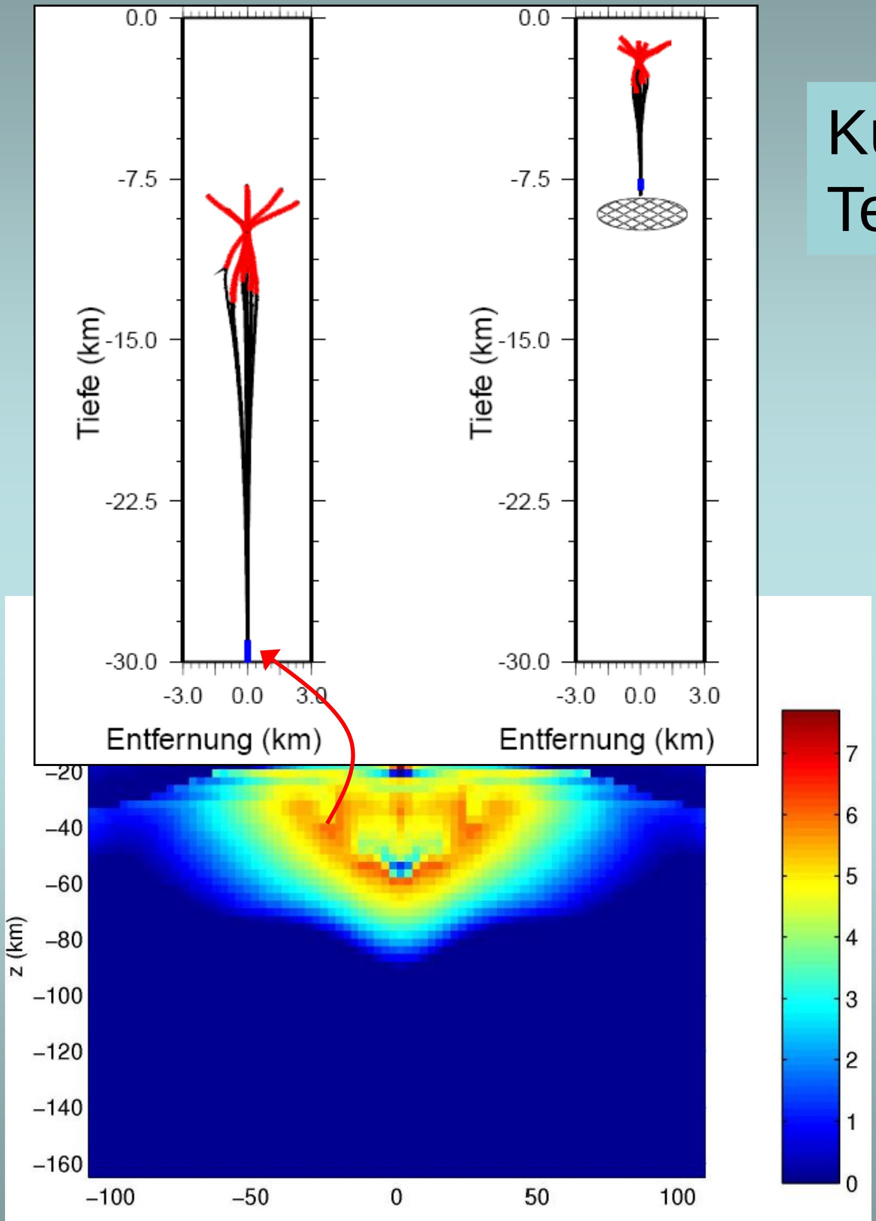
Snapshot, rising column of mantle with decompressional melting







Further ascent by magma driven propagating dykes

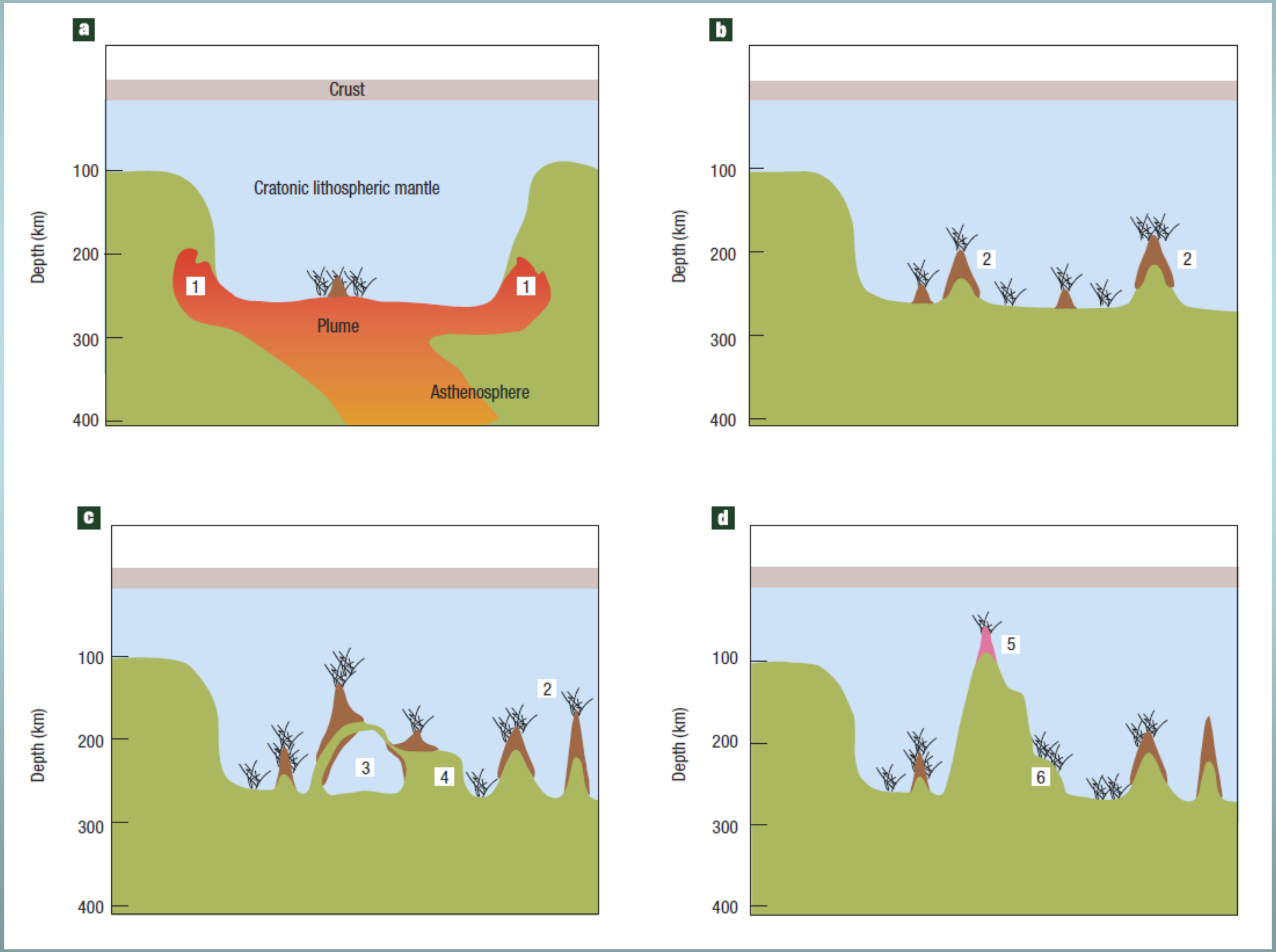


Kühn und Dahm, Tectonoph. 2006:

Do all of them make all the way through the cold lithosphere?

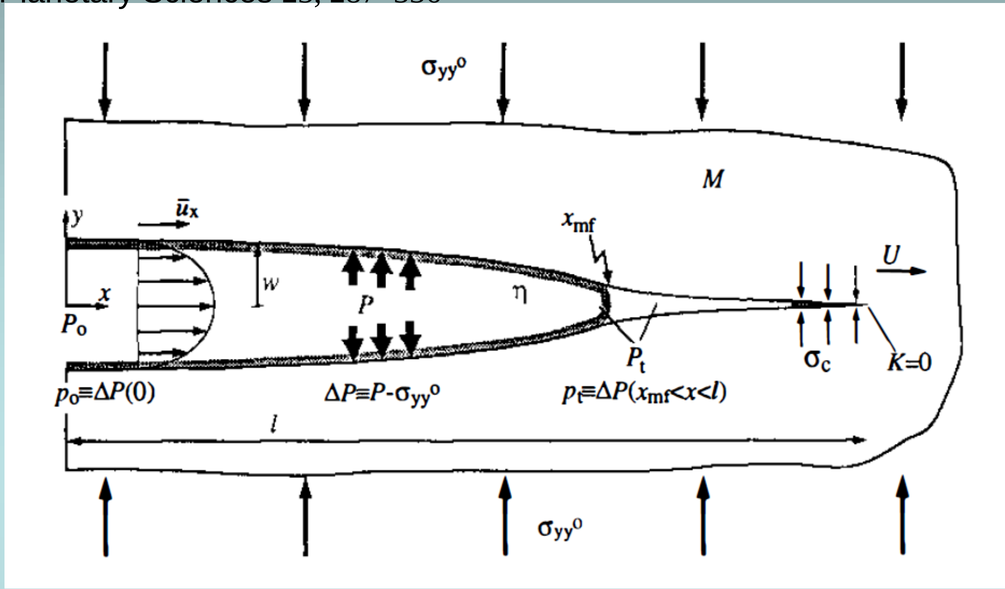
Magmatic impregnation of lithosphere. How does it work dynamically?

Foley 2008:



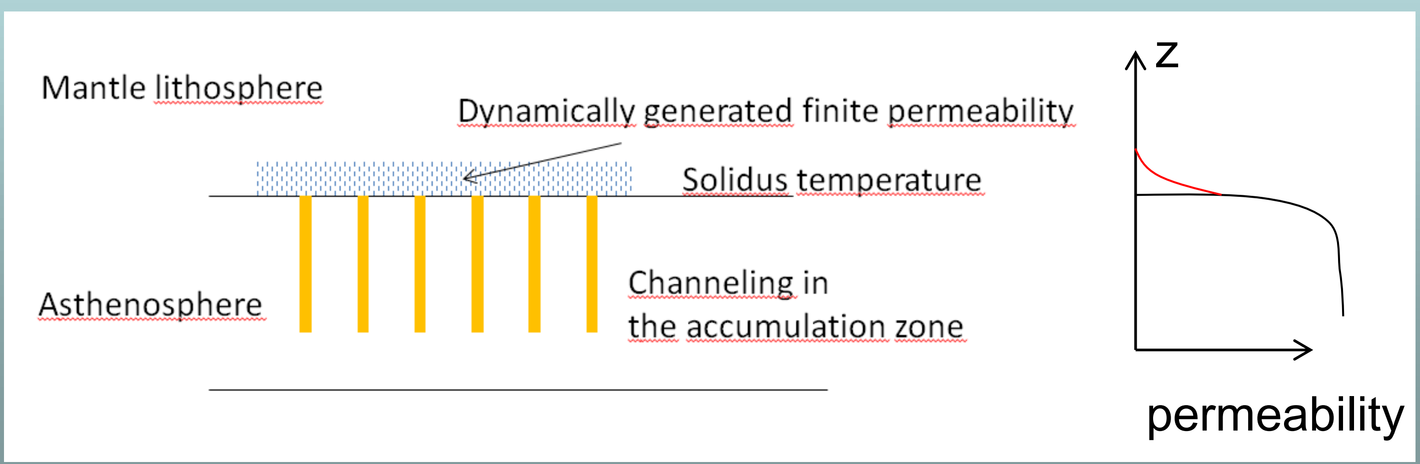
How to infiltrate melt into subsolidus base of lithosphere?

Rubin, A. M. (1995). Propagation of magma-filled cracks. Annual Review of Earth and Planetary Sciences 23, 287–336



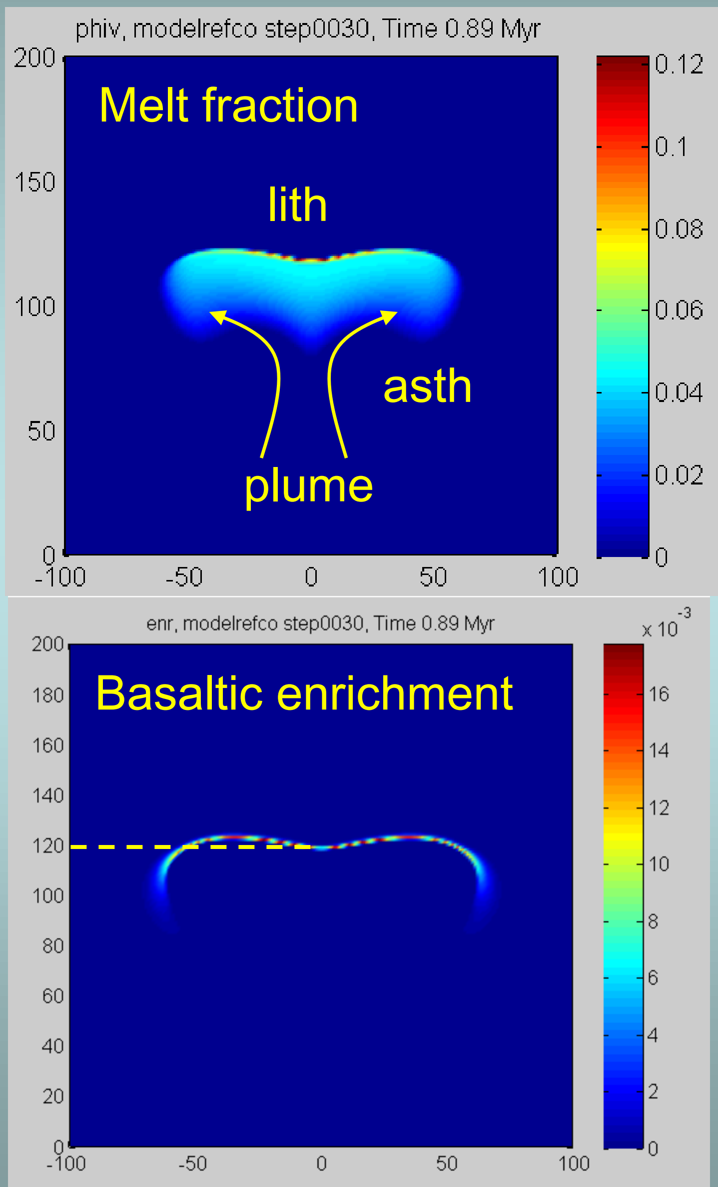
- Melt front opens a tensile crack „tip cavity"

→ Accumulated melt may encounter finite permeability at subsolidus conditions

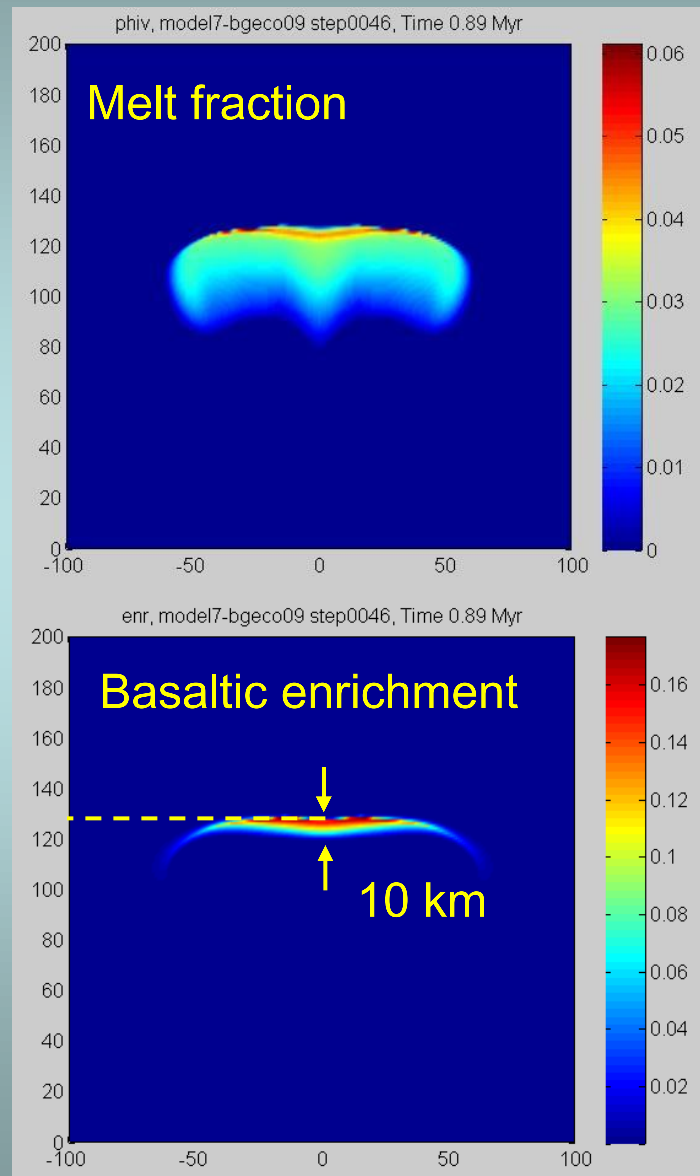


Sublithospheric plume

without



with finite subsolidus permeability

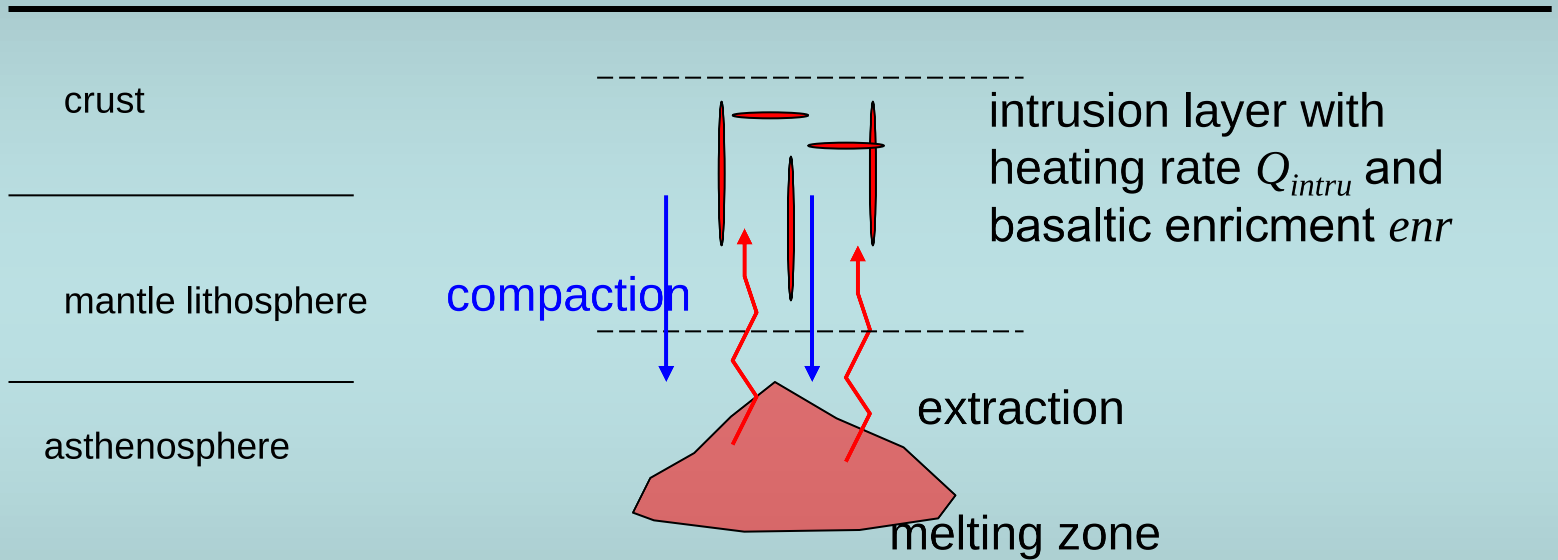


At this snapshot:

- 10km basaltic infiltration into lith base
- Infiltration controlled by freezing and latent heat

Alternative approach of melt extraction

Artificial extraction - intrusion (→ lithospheric weakening)

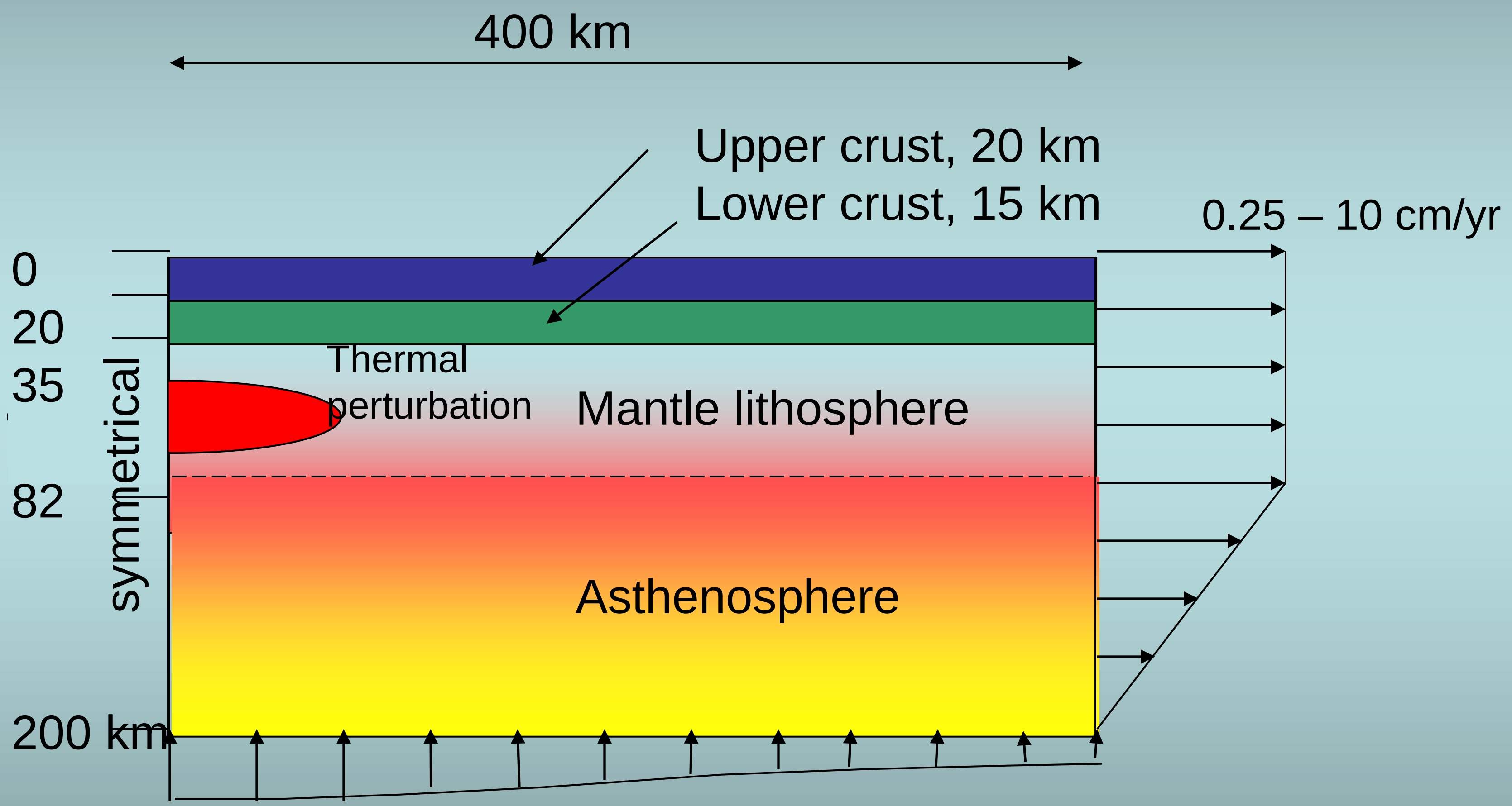


(intrusions not explicititly modelled)

$$Q_{intru} = \rho c_p \left(T_{source} - T_{ambient} + \left(1 - f_m(T_{ambient})\right) \cdot \frac{L}{c_p} \right) \cdot q_{intru}$$

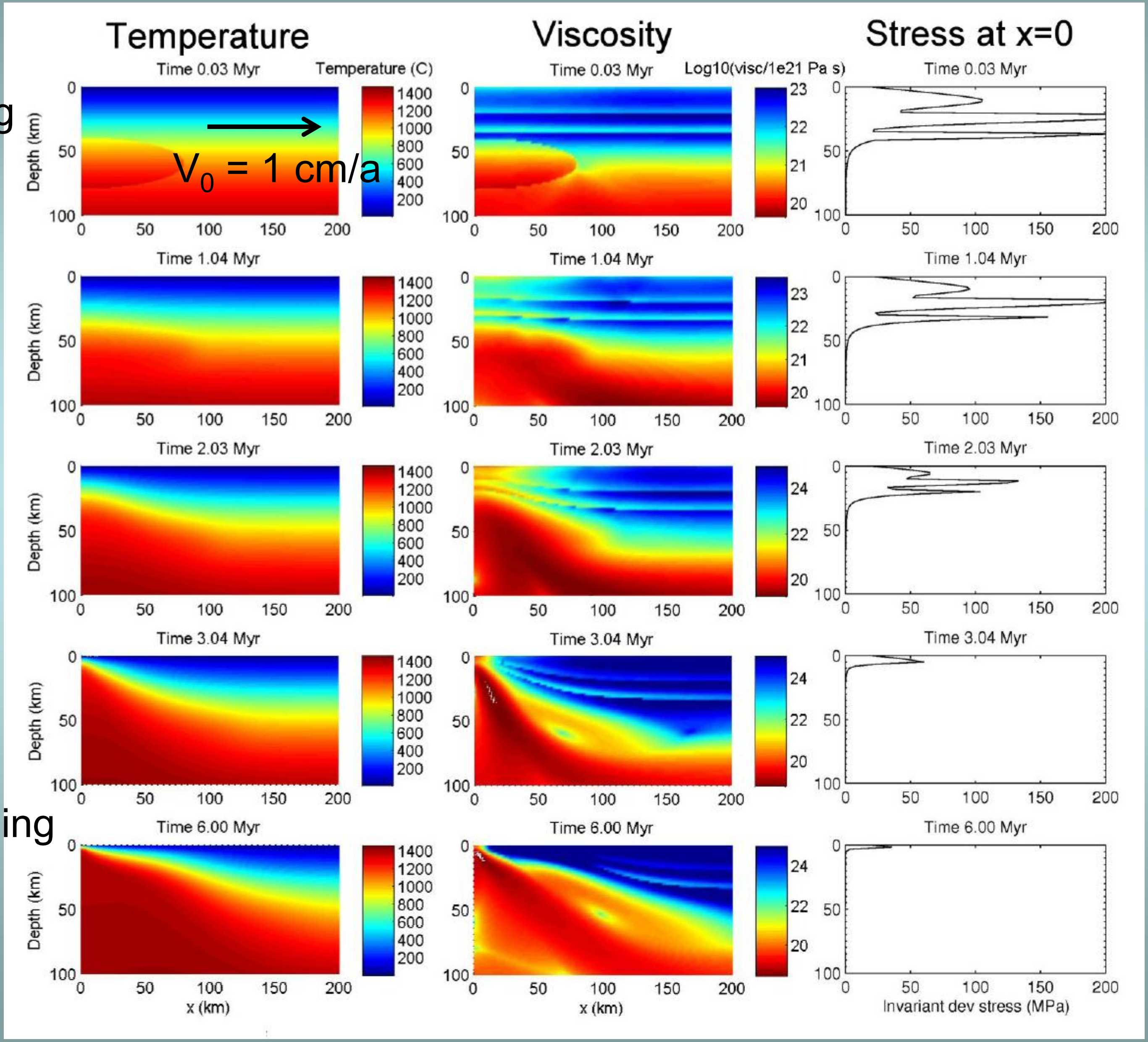
q_{intru} – volumetric intrusion rate

Applied to continental rifting and break up with melt extraction (Schmeling, 2009)



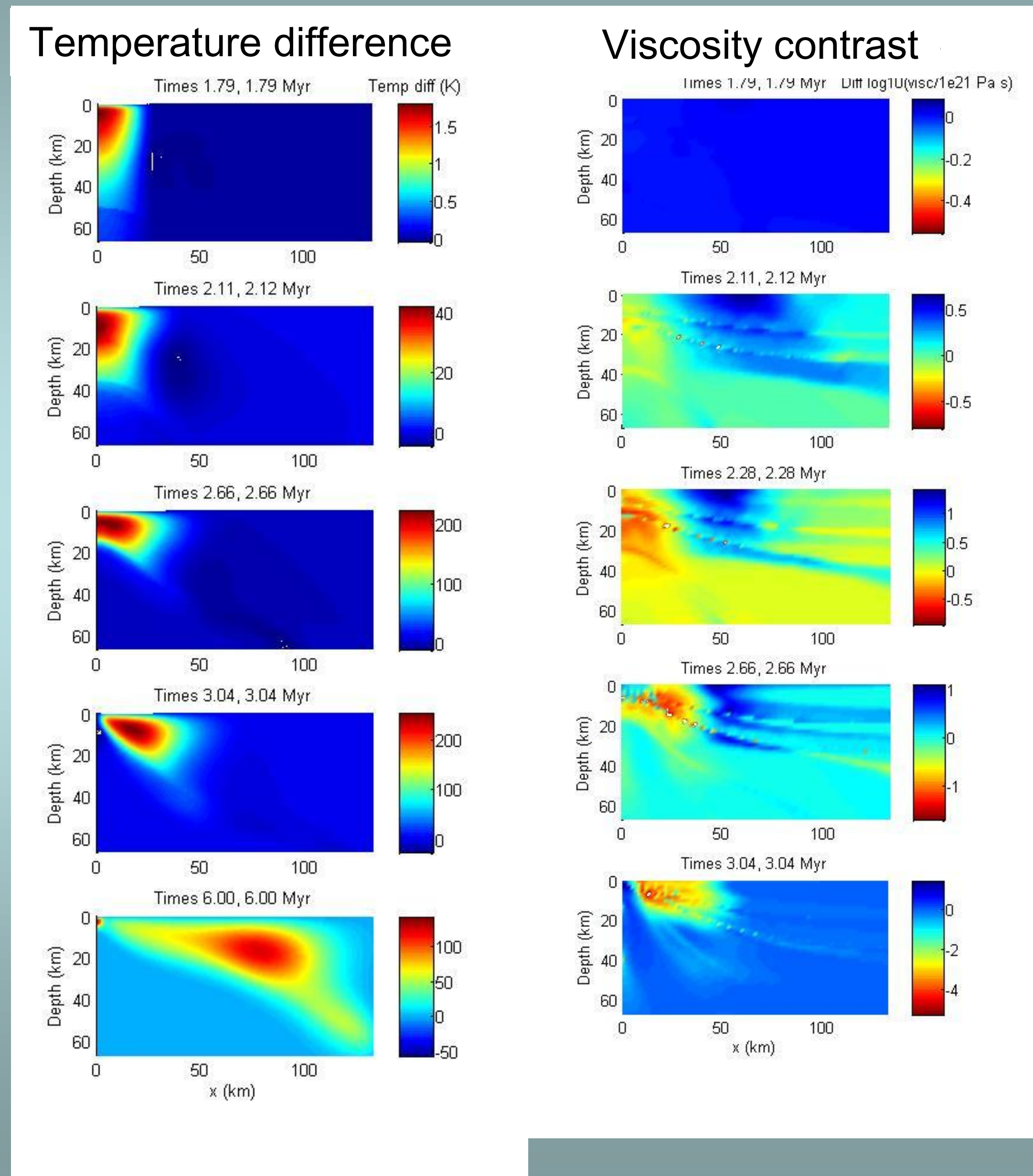
Continental rifting

Oceanic spreeading



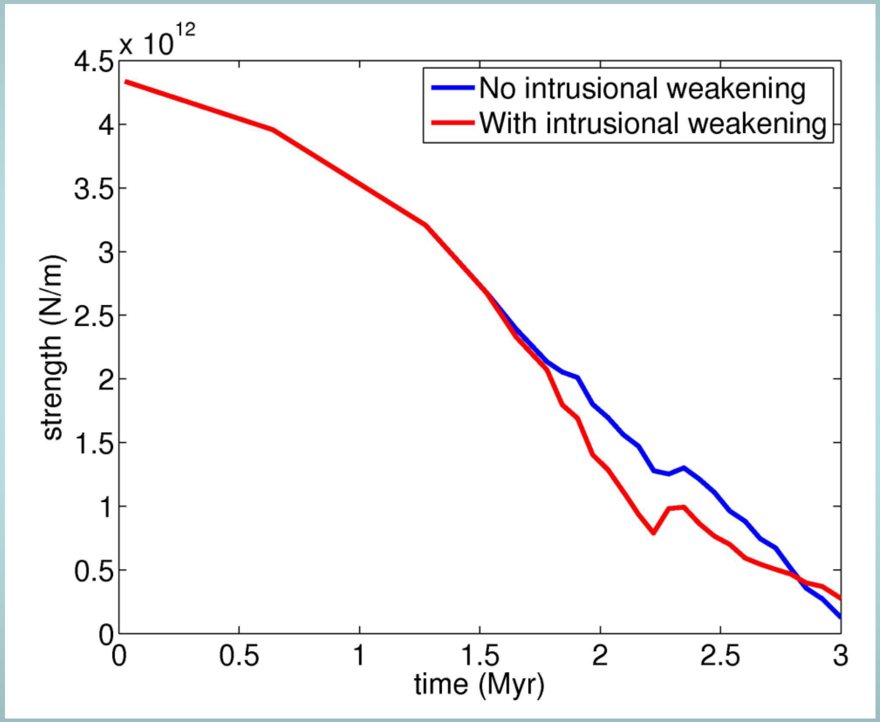
Effect of intrusional weakening

- Temperature increase by several 100 K
- Weakening: effective viscosity lower by up to one order of magnitude
- more effective melting (see below)

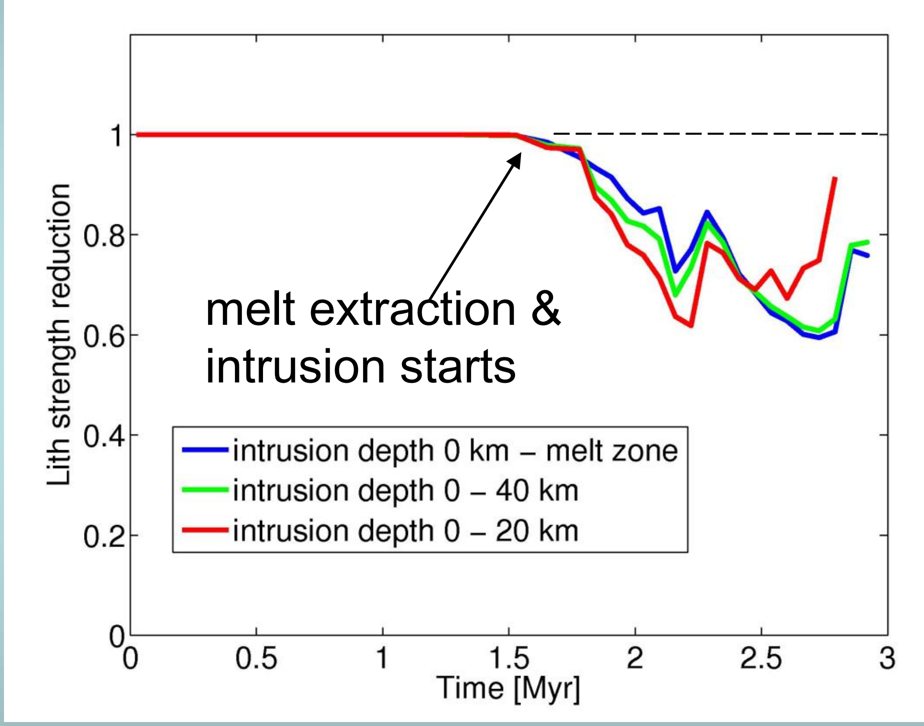


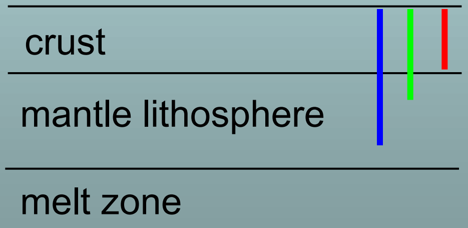
Effect of intrusional weakening

Reduction of lithospheric strength



Effect of emplacement depth





Conclusions

- Several two-phase flow melt – matrix formulations
- Effective shear and bulk viscosity is important, $f(\varphi)$ to be derived from melt inclusion models
- $\eta_s \sim (1-c_1\ \varphi)$, $\eta_b \sim (1-c_2\ \varphi)/\varphi$, η_b/η_s may drop for large φ
- Irrotational compaction flow may be handled like a load vector, =0 for const viscosity
- CBA works well
- Porosity dependent shear viscosity focuses melt flow and → channel instability
- Mantle: Melt accumulation near solidus
- Magma focusing at mid-ocen ridges controlled by high bulk viscosity
- Channel instability → effective anisotropic permeability
- Melt extraction models:
  - Critical porosities
  - Melt infiltration at lith base by tip-cavity permeability?
  - Melt weakening assistes rifting