12^{th} International Workshop on Modeling of Mantle Convection
and Lithospheric Dynamics

August 20^{th} to 25^{th} 2011, Döllnsee Germany

Two numerical methods to simulate flows with temperature dependent viscosity in the spherical shell

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The numerical solution of the Navier-Stokes equations in the spherical annulus is of fundamental interest for geophysical and astrophysical applications. The highly non-linear nature of the equations needs therefore special treatment. Consequently the full problem has to be calculated with numerical simulations. Due to the special geometry of the problem the class of spectral methods has become an appropriate tool to solve the equations in Boussinesq approximation. The presented method has been implemented into the numerical code developed by R. Hollerbach [1]. This code follows a spectral method, where the radial components are discretised on Chebyshev grid points. The toroidal-azimuthal components are further expanded as spherical harmonics. In a typical manner the boundaries are either stress-free or no-slip at the inner and outer sphere.

The viscous components are solved with an operator splitting method. In the first step the iso-viscous, linear terms are calculated in spectral space implicitly, which guarantees high numerical stability. The temperature dependent non-linear variations are transformed in real space, followed by a real-space calculation of the tensor elements and transformed back into spectral space. Viscosity contrasts of up to $\Delta\eta_T = 2$ can be calculated with comparable iso-viscous numerical resolutions. The reference values are Pr=185 and Ra=20000, which correspond to the working fluid 1-nonanol at 20°C and a temperature difference between the inner and the outer shell of 10°C. This physical setup is the reference state of the GeoFlowII experiment, which is mounted at the International Space Station (ISS) and currently running (July 2011).

In order to compare iso-viscous flows and simulations with temperature dependent viscosity the thermal Nusselt number is used. Global structures do not differ much, but the plumes show different widths in the temperature-dependent viscosity case. This process can be visualised with an interferometry method. The results are also compared with simulations done using the spherical mantle convection code GAIA [2], which solves the conservation equations of thermal convection for an incompressible Boussinesq fluid with infinite Prandtl number. The discretization of the governing equations is based on the finite-volume method with the advantage of using fully irregular grids [3]. GAIA uses the SIMPLE method to solve the coupling of the continuity equation with the momentum equation. The code can handle viscosity variations of up to 8 orders of magnitude from cell-to-cell and up to 45 orders of magnitude system wide. Benchmark comparison to published results and the comparison to the commercial software COMSOL Multiphysics© 3.5 yield satisfying results [4].

In a first step we compare the two codes for an iso-viscous fluid. The goal of this comparison was to investigate the Prandtl number influence, since for both the R. Hollerbach code and the GeoFlow Experiment Pr=64.64, whereas the GAIA code assumes $Pr \to \infty$.

In consequence of the numerical tests shown in Figure 1, the inertia forces can be dropped for Pr=64.64 or higher values.

Further, the effects of a fluid with temperature-dependent viscosity having a viscosity contrast of $\Delta\eta_T = 2$ are investigated. Although preliminary results from the GeoFlowII experiment show

substantial differences between iso-viscous and temperature-dependent viscosity cases, these differences cannot be reproduced with the two numerical codes. The results of the numerical tests show similar global structures and Nusselt numbers in both iso-viscous and temperature-dependent viscosity cases.
Further tests need to be done in order to better quantify the differences between the results obtained with the GeoFlowII experiment and the numerical simulations with temperature-dependent viscosity.

References

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Nusselt Number

	Axisymmetric (4,4)	Octahedral (3,2)	Cubic (random)
GeoFlow I	1,685	1,726	1,726
GAIA	1,690	1,727	1,728
GAIA, VC=2	1,372	1,377	1,392

Nusselt Number obtained with GeoFlow I and with GAIA code

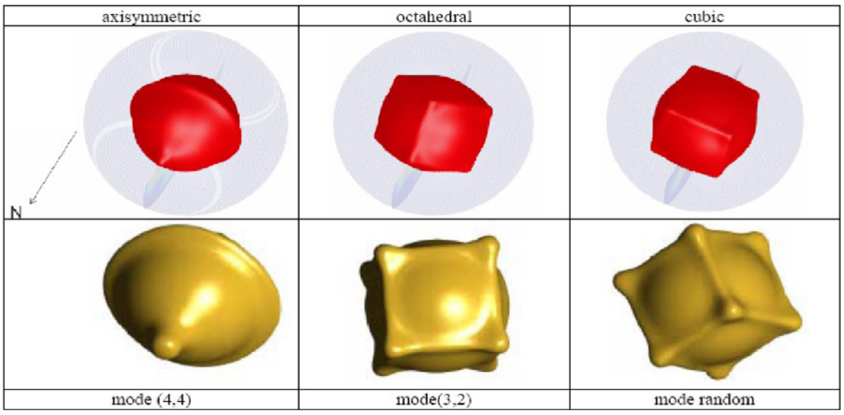

Figure 1: Numerical benchmark with R. Hollerbach code (top row) with Pr=64.64 and GAIA (bottom row) using $Pr = \infty$ for an iso-viscous fluid.