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Dual Reciprocity Boundary Element Method for studying thermal flow in cooling Magma Oceans

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Earth's early history is marked by a giant impact with a Mars-sized object with lead to the formation of the moon [1]. This impact event led to a substantial amount of melting of the Earth's interior. Subsequent cooling of the Earth involved extensive crystallization in this "magma ocean" over a relatively short period of time. While chemical evidence from ancient sources provides some clues on the rate of cooling, computational models of such phenomena are sparse.

Modeling the crystal settling behavior requires solving a coupled system of partial differential equations, specifically the Stokes flow equation coupled with the heat equation through an advection term. When solving such PDEs it is standard practice to use a finite element method (FEM) solver. FEM algorithms are very robust and generally converge with reasonable amounts of computation. Unfortunately, the settling of crystals make FEM impractical. As the crystals advect, the region of flow changes. The problem's dynamic geometry means the domain would have to be rediscretized at each time step. Rediscretization becomes extremely computationally costly, especially in high dimensions.

There exists standard techniques for solving multiparticle Stokes flow using boundary element method (BEM) [2]. BEM only requires discretization of the boundary of the particle. The lack of domain discretization allows the particle to advect without serious computational penalty. Unfortunately BEM requires knowledge of the fundamental solution to the PDE's adjoint operator, limiting it's usefulness to solve generic PDEs.

Our work applies the dual reciprocity boundary element method (DRBEM) introduced by Nardini and Brebbia [3][4] to solve the heat equation for the multiparticle system while avoiding discretization of the domain. DRBEM works by approximating parts of the PDE that would be troublesome to use in BEM by a linear combination of radial basis functions chosen a priori. Using the approximation of the residual portion allows for the boundary method to be applied to more complicated PDEs such as the heat equation.

References

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