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A parallel direct-iterative solver using domain decomposition approach for geodynamic modeling

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We present a parallel program for the solution of large linear systems resulting from the discretization of Partial Differential Equations that are particularly suitable for geodynamic modeling. Our approach is based on the *Schur* domain decomposition method [1], which allows efficient parallelization. It consists in partitioning the computational domain into non-overlapping sub-domains and a reordering of the nodes, allowing each sub-domain and the interface unknowns to be decoupled. The later, referred as *Schur* system, is solved either by an iterative or by a direct method. Then, each sub-domain set of unknowns is solved via block backward substitution. The parallelization is achieved by assigning the computations related to the different sub-domains to different sets of processors.

Our performance tests show that for two-dimensional problems it is better to solve the *Schur* system by a direct (Gaussian) method, whereas for three-dimensional problems we use a bi-conjugate gradient stabilized method [6] to solve efficiently the *Schur* system. The most part of the CPU time is spent in the computation of the *Schur* complement, which is done with a function in the MUMPS library [5]. An alternative algorithm to compute the complement from the *LU* factors is also being developed.

We have applied our solver to the Laplace/Poisson equations, the Tricomi equation and the biharmonic equation in 2D and 3D Cartesian domains of various aspect ratios. In all test cases the *Schur* approach performs better than the direct application of the MUMPS solver.

References

[1] J. S. Przemieniecki (1963), Matrix structural analysis of substructures, AIAAJ. 1, pp. 138-147
[2] P. E. Bjørstad, O. B. Widlund (1986), Iterative Methods for the Solution of Elliptic Problems on Regions Partitioned into Substructures, SIAM J. Numer. Anal. 23, pp. 1097-1120
[3] D. Vanderstraeten, R. Keunings, A parallel solver based on the dual *Schur* decomposition of general finite element matrices, International Journal for Numerical Methods in Fluids 28, (1998) pp. 23-46
[4] L. Mansfield (1990), On the Conjugate Gradient Solution of the *Schur* Complement System Obtained from Domain Decomposition, SIAM J. Numer. Anal. 27, pp. 1612-1620
[5] P. R. Amestoy, I. S. Duff , J . Koster, J.-Y. L'Excellent (2001), A fully asynchronous multifrontal solver using distributed dynamic scheduling, SIAM Journal of Matrix Analysis and Applications 23, pp. 15-41
[6] D. R. Fokkema, Enhanced implementation of BiCGStab(l) for solving linear systems of equations. Preprint 976, Department of Mathematics, Utrecht University.