Klausur 13.02.09 | 3 h | 1

1.

(a)

$$A = \begin{pmatrix} 2 & 1 & 0 & q \\ 4 & 3 & 2 & 0 \\ 0 & 2 & 3 & 1 \\ 2 & 0 & -2 & 3 \end{pmatrix} \qquad q \in R$$

$$\boxed{Ax = b}$$

$$b = \begin{pmatrix} 2 \\ 2 \\ 0 \\ c \end{pmatrix} \qquad c \in R$$

$$\det A = 2 \cdot (+1) \begin{vmatrix} 3 & 2 & 0 \\ 2 & 3 & 1 \\ 0 & -2 & 3 \end{vmatrix} + 1 \cdot (-1) \begin{vmatrix} 4 & 2 & 0 \\ 0 & 3 & 1 \\ 2 & -2 & 3 \end{vmatrix} + 0 \cdot (+1) + q \cdot (-1) \begin{vmatrix} 4 & 3 & 2 \\ 0 & 2 & 3 \\ 2 & 0 & -2 \end{vmatrix}$$

Entwicklung nach 1. Zeile

$$= 2 \{ 27 - [-6 + 12] \}$$
$$- 1 \{ 36 + 4 - [-8] \}$$
$$- q \{ -16 + 18 - [8] \}$$
$$= 42 - 48 + 6q = \underline{-6 + 6q}$$

(b)

$$\mathrm{rg}(A) = \begin{cases} 4 & q \neq 1 \\ 3 & q = 1 \end{cases}$$

(C) für $q \neq 1$ eindeutig lösbar

Lösung über Gauß Elimination

$$\begin{pmatrix} 2 & 1 & 0 & q \\ 4 & 3 & 2 & 0 \\ 0 & 2 & 3 & 1 \\ 2 & 0 & -2 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \\ c \end{pmatrix}$$

$$\begin{array}{l} \\ -2\times \text{Zeile } 1 \\ \\ -1\times \text{Zeile } 1 \end{array} \begin{pmatrix} 2 & 1 & 0 & q \\ 0 & 1 & 2 & -2q \\ 0 & 2 & 3 & 1 \\ 0 & -1 & -2 & 3-q \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 0 \\ c-2 \end{pmatrix}$$

$$\begin{array}{l} \\ \\ -2\times \text{Zeile } 2 \\ +1\times \text{Zeile } 2 \end{array} \begin{pmatrix} 2 & 1 & 0 & q \\ 0 & 1 & 2 & -2q \\ 0 & 0 & -1 & 1+4q \\ 0 & 0 & 0 & 3-3q \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 4 \\ c-4 \end{pmatrix}$$

$$\rightsquigarrow \boxed{x_4 = \frac{c-4}{3-3q}} \underset{c=1+3q}{=} \frac{-3+3q}{3-3q}$$

$$(-1)\cdot x_3 + (1+4q)\,x_4 = 4$$

$$\boxed{x_3 = \frac{(1+4q)(c-4)}{3(1-q)} - 4}$$

$$x_2 + 2x_3 - 2q x_4 = -2$$

$$x_2 = -2 - \frac{2(1+4q)(c-4)}{3(1-q)} + 8 + 2q\,\frac{c-4}{3(1-q)}$$

$$x_2 = 6 + \frac{-2(1+4q)(c-4) + 2q(c-4)}{3(1-q)}$$

$$x_2 = \frac{18(1-q) + (c-4)[-2(1+4q) + 2q]}{3(1-q)}$$

$$\boxed{x_2 = \frac{18(1-q) + (c-4)(-2-6q)}{3(1-q)}}$$

$$2x_1 + x_2 + q x_4 = 2$$

$$x_1 = 1 - \frac{1}{2}x_2 - \frac{q}{2}x_4$$

$$= 1 - \frac{9(1-q) + (c-4)(-1-3q) - \frac{1}{2}(c-4)}{3(1-q)}$$

Setzen $c = 1+3q$

$$x_4 = \frac{1+3q-4}{3-3q} = \frac{-3+3q}{3-3q} = \boxed{-1}$$

$$(-1)x_3 + (1+4q)x_4 = 4$$

$$(-1)x_3 - (1+4q) = 4 \qquad x_3 = -4-(1+4q)$$

$$\boxed{x_3 = -5-4q}$$

$$x_2 + 2x_3 - 2q x_4 = -2$$

$$x_2 + 2(-5-4q) + 2q = -2$$

$$x_2 = -2 + 10 + 6q \rightarrow \boxed{x_2 = 8+6q}$$

$$2x_1 + x_2 + q x_4 = 2$$

$$2x_1 + 8 + 6q - q = 2 \rightarrow \boxed{x_1 = -3 - \frac{5}{2}q}$$

Probe:

(1z) $2x_1 + x_2 + q x_4 = 2$

$+2(-3 - \frac{5}{2}q) + (8+6q) - q = 2$

$-6 - 5q + 8 + 6q - q = 2$

$2 = 2$ ✓

(2z) $4x_1 + 3x_2 + 2x_3 = 2$

$4(-3 - \frac{5}{2}q) + 3(8+6q) + 2(-5-4q) = 2$

$-12 - \cancel{10q} + 24 + \cancel{18q} - 10 - \cancel{8q} = 2$ ✓

(3z) $2x_2 + 3x_3 + x_4 = 0$

$2(8+6q) + 3(-5-4q) - 1 = 0$ ✓

$16 + 12q - 15 - 12q - 1 = 0$

(4z) $2x_1 - 2x_3 + 3x_4 = 1 + 3q$

$2(-3 - \frac{5}{2}q) - 2(-5-4q) - 3 = 1 + 3q$

$-6 - 5q + 10 + 8q - 3 = 1 + 3q$ ✓

Gleichungssystem für $q=2$, $c=1$ lösen (5)
$\neq 1+3q$!

$$A = \begin{pmatrix} 2 & 1 & 0 & 2 \\ 4 & 3 & 2 & 0 \\ 0 & 2 & 3 & 1 \\ 2 & 0 & -2 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{array}{l} \\ -2 * \text{Zeile } 1 \\ \\ -1 * \text{Zeile } 1 \end{array} \begin{pmatrix} 2 & 1 & 0 & 2 \\ 0 & 1 & 2 & -4 \\ 0 & 2 & 3 & 1 \\ 0 & -1 & -2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 0 \\ -1 \end{pmatrix}$$

$$\begin{array}{l} \\ \\ -2 \times \text{Zeile } 2 \\ +1 \times \text{Zeile } 2 \end{array} \begin{pmatrix} 2 & 1 & 0 & 2 \\ 0 & 1 & 2 & -4 \\ 0 & 0 & -1 & 9 \\ 0 & 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 4 \\ -3 \end{pmatrix}$$

$\rightsquigarrow \underline{x_4 = +1}$

$-x_3 + 9x_4 = 4 \quad \rightarrow \quad \underline{x_3 = 5}$

$x_2 + 2x_3 - 4x_4 = -2 \quad \rightarrow \quad x_2 = -2 - 10 + 4$

$\underline{x_2 = -8}$

$2x_1 + x_2 + 2x_4 = 2 \quad \rightsquigarrow \quad 2x_1 = 2 - x_2 - 2x_4$

$= 2 + 8 - 2 = 8$

$\underline{x_1 = 4}$

$$\boxed{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 4 \\ -8 \\ 5 \\ 1 \end{pmatrix}}$$

$\boxed{\text{Probe}}$

$2 \cdot 4 + (-8) + 2 \cdot 1 = 2$ ✓

$4 \cdot 4 + 3(-8) + 2 \cdot 5 = 2$ ✓

$2(-8) + 3 \cdot 5 + 1 \cdot 1 = 0$ ✓

$2 \cdot 4 - 2 \cdot 5 + 3 \cdot 1 = 1$ ✓

(1)(d) Fall der nicht eindeutigen Lösbarkeit

$\boxed{q=1}$

Für welche c existiert keine Lösung?

$$\rightsquigarrow \underbrace{Rg(A)}_{=3} \neq \underbrace{Rg\{A;b\}}_{=4 \text{ falls ungleich}}$$

$$\begin{vmatrix} 2 & 1 & 0 & 2 \\ 4 & 3 & 2 & 2 \\ 0 & 2 & 3 & 0 \\ 2 & 0 & -2 & c \end{vmatrix} = \begin{cases} \neq 0 & \text{lin. unabh., d.h. } Rg\{A;b\}=4 \\ = 0 & \text{lin. abh., d.h. } Rg\{A;b\}=3 \end{cases}$$

eigtl. vorher $Rg\{A\}$ bestimmen!

$$\begin{pmatrix} 2 & 1 & 0 & 1 \\ 4 & 3 & 2 & 0 \\ 0 & 2 & 3 & 1 \\ 2 & 0 & -2 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & 0 & 1 \\ 0 & 1 & 2 & -2 \\ 0 & 2 & 3 & 1 \\ 0 & -1 & -2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & 0 & 1 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & -1 & 5 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad Rg\{A\}=3$$

$\boxed{x_4 = \text{bel.} = t} \rightarrow \boxed{x_3 = 5t}$

$x_2 = 2t - 10t \boxed{= -8t}$

$2x_1 = -x_2 - x_4 = 8t - t$

$\boxed{x_1 = 7/2\, t}$

Determinante berechnen von $\{A;b\}$

$$\begin{vmatrix} 2 & 1 & 0 & 2 \\ 4 & 3 & 2 & 2 \\ 0 & 2 & 3 & 0 \\ 2 & 0 & -2 & c \end{vmatrix}$$

Entw. nach 3. Zeile

$$= 2\cdot(-1)\begin{vmatrix} 2 & 0 & 2 \\ 4 & 2 & 2 \\ 2 & -2 & c \end{vmatrix}$$

$$+3\cdot(+1)\begin{vmatrix} 2 & 1 & 2 \\ 4 & 3 & 2 \\ 2 & 0 & c \end{vmatrix}$$

$$= (-2)\{4\cdot c - 16 - [8-8]\}$$

$$+(+3)\{6\cdot c + 4 - [12 + 4c]\}$$

$$= -8c + 32 + 6c - 24 = \underline{-2c+8}$$

Für $\boxed{c=4}$ ist $\det\{A;b\} = 0$

$\hookrightarrow$ d.h. dann Vektoren lin. abhängig ; $s = n - Rg(A)$

$\hookrightarrow$ exist. unendl. viele Lösungen !
mit dim{Lösungsraum} = 1

Für $\boxed{c \neq 4}$ ist $\det\{A;b\} \neq 0$

$\hookrightarrow$ exist. <u>keine</u> Lösung

Aufgabe 2

$v_i \in \mathbb{R}^4$
$i = 1, 2, 3$

$$\begin{pmatrix} 1 \\ 2 \\ -3 \\ -4 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ 3 \\ -2 \end{pmatrix} \begin{pmatrix} a \\ 4 \\ -6 \\ 1 \end{pmatrix} \quad a \in \mathbb{R}$$

a) für welche a lin. unabh./abh. ?

$$\lambda_1 v_1 + \lambda_2 v_2 + \lambda_3 v_3 = 0 \quad \rightarrow \lambda_1 = \lambda_2 = \lambda_3 = 0 ?$$

$$\begin{pmatrix} 1 \\ 2 \\ -3 \\ -4 \end{pmatrix} \lambda_1 + \begin{pmatrix} 1 \\ -2 \\ 3 \\ -2 \end{pmatrix} \lambda_2 + \begin{pmatrix} a \\ 4 \\ -6 \\ 1 \end{pmatrix} \lambda_3 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & +1 & a \\ 2 & -2 & 4 \\ -3 & 3 & -6 \\ -4 & -2 & 1 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

lösen!

$$\begin{array}{l} \\ -2 \times \text{Zeile } 1 \\ +3 \cdot \text{Zeile } 1 \\ +4 \cdot \text{Zeile } 1 \end{array} \begin{pmatrix} 1 & 1 & a \\ 0 & -4 & 4-2a \\ 0 & 6 & -6+3a \\ 0 & +2 & 1+4a \end{pmatrix} \begin{pmatrix} \\ \\ \\ \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & a \\ 0 & -4 & 4-2a \\ 0 & 6 & -6+3a \\ 0 & 2 & 1+4a \end{pmatrix} \begin{pmatrix} \\ \\ \\ \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{array}{r} \\ \\ +\frac{3}{2} * \text{Zeile } 2 \\ +\frac{1}{2} * \text{Zeile } 2 \end{array} \begin{pmatrix} 1 & 1 & a \\ 0 & -4 & 4-2a \\ 0 & 0 & 0 \\ 0 & 0 & 3+3a \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\lambda_3 = 0 \rightsquigarrow \boxed{a \neq -1}$$

Falls $a = -1$ folgt

$$\begin{pmatrix} 1 & 1 & -1 \\ 0 & -4 & 6 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left.\begin{aligned} \lambda_1 + \lambda_2 - \lambda_3 &= 0 \\ -4\lambda_2 + 6\lambda_3 &= 0 \end{aligned}\right\} \quad \begin{aligned} \lambda_2 &= \tfrac{3}{2}\lambda_3 \\ \lambda_1 &= -\tfrac{1}{2}\lambda_3 \end{aligned}$$

$\exists$ nicht-triviale Lösung

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = \begin{pmatrix} -t \\ +3t \\ +2t \end{pmatrix}$$

Vektoren lin. abhängig

Falls $a \neq -1$ folgt

$$\boxed{\begin{aligned} \lambda_3 &= 0 \\ \lambda_2 &= 0 \\ \lambda_1 &= 0 \end{aligned}}$$

Vektoren linear unabhängig

(brauchen (unabh. von a) 2 Vektoren für Erz.system)

$$\dim\{U\} = \begin{cases} 2 & \text{für } a = -1 \\ 3 & \text{für } a \neq -1 \end{cases}$$

Aufgabe 3

Abb. $\varphi: \mathbb{R}^4 \to \mathbb{R}^2$

$$\varphi[x] = \varphi\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} a x_1 + b x_3 \\ x_2 - x_4 \end{pmatrix}$$

$a \neq 0$
$b \neq 0$

a)
z.z. φ ist linear

$$\varphi[x+y] = \varphi[x] + \varphi[y]$$
$$\varphi[\lambda x] = \lambda \varphi[x]$$

$$\varphi[x+y] = \varphi\begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \\ x_3 + y_3 \\ x_4 + y_4 \end{pmatrix} = \begin{pmatrix} a(x_1+y_1) + b(x_3+y_3) \\ x_2+y_2 - (x_4+y_4) \end{pmatrix}$$

$$= \begin{pmatrix} a x_1 + b x_3 \\ x_2 - x_4 \end{pmatrix} + \begin{pmatrix} a y_1 + b y_3 \\ y_2 - y_4 \end{pmatrix}$$

$$= \varphi[x] + \varphi[y]$$

$$\varphi[\lambda x] = \begin{pmatrix} \lambda a x_1 + \lambda b x_3 \\ \lambda x_2 - \lambda x_4 \end{pmatrix} = \lambda \begin{pmatrix} a x_1 + b x_3 \\ x_2 - x_4 \end{pmatrix} = \lambda \varphi[x]$$

Abb.-Matrix

$$\varphi[x] = Ax = \begin{pmatrix} a x_1 + b x_3 \\ x_2 - x_4 \end{pmatrix} = \begin{pmatrix} a & 0 & b & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

(2×4) Matrix

(c) kern der Abb A

$$A \cdot x = 0$$

$$a x_1 + b x_3 = 0 \qquad x_1 = -\frac{b}{a} x_3 \text{ bel.}$$

$$x_2 - x_4 = 0 \qquad x_2 = x_4 \text{ bel.}$$

$$\ker\{A\} = \begin{pmatrix} -\frac{b}{a} t_1 \\ t_2 \\ t_1 \\ t_2 \end{pmatrix} \qquad \mathrm{Dim}\{\ker\} = 2$$

Aufgabe 4

(a) $y'' - 6y' + 13y = 13x + 7$

homog. Lösung

$y(x) = A e^{\lambda x}$

$y'(x) = A \cdot \lambda e^{\lambda x}$

$y''(x) = A \cdot \lambda^2 e^{\lambda x}$

char. Polynom

$\lambda^2 - 6\lambda + 13 = 0$

$\lambda_{1/2} = +3 \pm \sqrt{9-13}$ ⇝ oszill. Lösung

$= +3 \pm 2i$

Allg. Lösung

$$\boxed{\begin{aligned} y_h(x) &= C_1 e^{3x} \cos 2x + C_2 e^{3x} \sin 2x \\ &= e^{3x}[C_1 \cos 2x + C_2 \sin 2x] \end{aligned}}$$

Probe: $y' = 3e^{3x}[C_1 \cos 2x + C_2 \sin 2x]$
$\quad + e^{3x}[-2C_1 \sin 2x + 2C_2 \cos 2x]$

$y'' = 9e^{3x}[C_1 \cos 2x + C_2 \sin 2x]$
$\quad + 6e^{3x}[-2C_1 \sin 2x + 2C_2 \cos 2x]$
$\quad + e^{3x}[-4C_1 \cos 2x - 4C_2 \sin 2x]$

$-6y' = -18e^{3x}[C_1 \cos 2x + C_2 \sin 2x]$
$\quad - 6e^{3x}[-2C_1 \sin 2x + 2C_2 \cos 2x]$

$+13y = 13e^{3x}[C_1 \cos 2x + C_2 \sin 2x]$

$=$ ✓ ✓ ✓

Spezielle Lösung

$$\left.\begin{aligned} y &= x + a \\ y' &= 1 \\ y'' &= 0 \end{aligned}\right\}$$

$$-6 + 13(x+a) = 13x + 7$$
$$-6 + 13a = 7$$
$$13a = 13$$
$$a = 1$$

$$y_{sp} = x + 1$$

Insges.: $y(x) = e^{3x}[c_1 \cos 2x + c_2 \sin 2x] + (x+1)$

AB: $y(0) = 5$ $\rightsquigarrow y(0) = c_1 + 1 = 5$ $\boxed{c_1 = 4}$

$y'(0) = 3$

$$y'(x) = 3e^{3x}[c_1 \cos 2x + c_2 \sin 2x] + e^{3x}[-2c_1 \sin 2x + 2c_2 \cos 2x] + 1 = 3$$

$$y'(0) = 3\cancel{e^{3x}}c_1 + \cancel{e^{3x}}2c_2 + 1 = 3$$

$$3c_1 + 2c_2 + 1 = 3$$

$$12 + 2c_2 = 2 \rightsquigarrow \boxed{c_2 = -5}$$

$$\boxed{y(x) = e^{3x}[4\cos 2x - 5\sin 2x] + (x+1)}$$

(b)

$$y^{(4)} - \frac{4}{9} y'' = \frac{8}{3} x^2$$

ges.: Allg. Lösung

$y''(x) = z(x)$ $\qquad z'' - \frac{4}{9} z = \frac{8}{3} x^2$

homog. $\quad z_h(x) = C_1 e^{\frac{2}{3}x} + C_2 e^{-\frac{2}{3}x}$

Spez. inhom. $\quad z_{spez}(x) = A \cdot x^2 + B$

$z'' = 2A$

$2A - \frac{4}{9}(Ax^2 + B) = \frac{8}{3} x^2$

$\rightsquigarrow -\frac{4}{9} A = \frac{8}{3}$

$\underline{A = -6}$ //

$2A - \frac{4}{9} B = 0$

$-12 - \frac{4}{9} B = 0 \rightsquigarrow \underline{B = -27}$ //

$\underline{z_{spez}(x) = -6x^2 - 27}$ //

Allg. Lösung $\quad z(x) = C_1 e^{\frac{2}{3}x} + C_2 e^{-\frac{2}{3}x} - 6x^2 - 27$

$\Downarrow$

$$\boxed{y(x) = C_1' e^{\frac{2}{3}x} + C_2' e^{-\frac{2}{3}x} - \frac{1}{2} x^4 - \frac{27}{2} x^2 + C_3 x + C_4}$$

Aufgabe 5

$$\frac{dI(t)}{dt} = I(t)\frac{N-I(t)}{N}k$$

setze $y(t) := I(t)/N$

$$N \cdot \frac{dy}{dt} = N \cdot y(t)\,[1-y(t)]\,k$$

$$\frac{dy}{dt} = y(1-y)\,k$$

(a) Allg. Lösung $\rightsquigarrow$ Trennung der Variablen!

$$\frac{dy}{y(1-y)} = k\,dt$$

$$\frac{1}{y(1-y)} = \frac{1}{y} + \frac{1}{(1-y)}$$

$$\int \frac{dy}{y(1-y)} = \int \frac{dy}{y} + \int \frac{dy}{1-y}$$

$$= \ln y - \ln(1-y) = \ln\frac{y}{1-y}$$

$$\ln\frac{y}{1-y} = k \cdot t + \text{const.}$$

$$\frac{y}{1-y} = c \cdot e^{kt}$$

$$y = c \cdot e^{kt}\,[1-y]$$

$$y[1 + c \cdot e^{kt}] = c\,e^{kt}$$

$$y(t) = \frac{c_1 e^{kt}}{1 + c_1 e^{kt}} = \frac{1}{1 + c_2 e^{-kt}}$$

Probe :

$$\frac{dy}{dt} = \frac{+C_2 k e^{-kt}}{(1+C_2 e^{-kt})^2}$$

$$y(1-y)k = \frac{C_2 e^{-kt} k}{(1+C_2 e^{-kt})^2} \quad ✓$$

also

$$\boxed{I(t) = \frac{N}{1+C_2 \cdot e^{-kt}}}$$ allg. Lösung

(b)

AB : $I(0)=1$ berücksichtigen $\rightsquigarrow$ $I(0) = \frac{N}{1+C_2} = 1$

$1 + C_2 = N$

$C_2 = N - 1$

$$\boxed{I(t) = \frac{N}{1+(N-1)e^{-kt}}}$$

k ... Zeiteinheit $\sim$ "Tage"

(c) $N = 120000$ Einwohner von Potsdam

$I(3) = 20 \rightarrow I(3) = \frac{120\,000}{1+119999 e^{-3}}$

$\{1+(N-1)e^{-kt}\} I = N$

$(N-1)e^{-kt} I = N - I$

$(N-1)e^{-kt} = \frac{N-I}{(N-1)I}$

$I(x) = 60000$

$x = ?$ nach wieviel Tagen $I = 60000$?

$k \cdot 3 = \ln\left\{\frac{(N-1)I}{N-I}\right\}$

$k \cdot x = \ln\left\{\frac{119999 \cdot 60000}{60000}\right\}$

$k \cdot 3 = \ln\left\{\frac{119999 \cdot 20}{119980}\right\}$

Bestimmung von "k" $= \ln\left\{\frac{119999 \cdot 20}{119980}\right\}$

$x/3 = 11.695/2.996 \Rightarrow \underline{x = 11,71}$

$K = 0.99863$